

# MATH CHALLENGE PROBLEM

for September 2017

## Subsets of Subsets

$$A \subseteq B \subseteq [n]$$

Consider the set  $[n] = \{1, 2, 3, \dots, n\}$ . It is fairly easy to see that there are  $2^n$  different subsets  $B \subseteq [n]$ . For each such subset, we can also count the number of subsets  $A \subseteq B$ , which will be  $2^k$  where  $k$  is the size of  $B$ . We would like to combine these to count the number of pairs  $(A, B)$  such that  $A \subseteq B \subseteq [n]$ .

**The Challenge:** Find an elegant formula for the number of pairs  $(A, B)$  of subsets of  $[n]$  with  $A \subseteq B \subseteq [n]$ . Bonus, express this formula in as many ways as possible, and explain why they all make sense.

Submit solutions to Ross 2239G or [oscar.levin@unco.edu](mailto:oscar.levin@unco.edu) by **Friday, September 29**.

The best solution will WIN A PRIZE!

Prizes include nifty Rubik's style puzzle cubes, math puzzle books, math games, even a math coloring book. So submit your answer TODAY!