

## MED Comps: Summer 2011

### PART I

**ANSWER ALL** (3 hours with computer). This part of the exam focuses on general knowledge, mainly from coursework and readings in MED 610, 700, 701 and 702.

Provide responses to entire questions. All responses should be in essay form; aim for clarity and explicitness, as well as thoroughness, concision, and coherence in your writing. Define all technical terminology that you use in your responses and be explicit about how you are applying the ideas.

- Write your name on the first page of each response.
  - Start each answer on a new sheet of paper and double space the final version. If you use diagrams in your responses, label each with a title (e.g., “Figure x”) and insert a clear reference to each one in the appropriate place in your narrative.
  - In all cases, support assertions with citations from the literature, as appropriate, using APA format: (Author(s), Year).
  - **SAVE** your work on the hard-drive **OFTEN** and then save your final copy on the jump drive provided by the proctor. These will be sent to Dr. Soto, who will blind the students’ work.
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1. The abstract for a research report conducted by Bowd and Brady (2003) reads as follows:

*A survey of 357 final-year preservice teachers at a small Canadian university found that male and female participants did not differ in mathematics achievement or time elapsed since taking a mathematics course, but women had significantly greater mathematics anxiety and reported less positive beliefs about mathematics utility and more negative experiences with mathematics instruction in high school than men (Bowd & Brady, 2003).*

Bowd and Brady would like to extend this study through qualitative methods in order to provide possible explanations for these results. Provide an overview of a proposal for a qualitative study. As part of this overview address major issues of educational research, including

- a. the research question(s),

- b. relevant literature,
- c. a description of the setting and participants,
- d. the proposed method(s) of data collection and analysis methods, including ways to ensure valid and reliable results, and
- e. the limitations of your proposed study.

2. The program officer of a project supported by the National Science Foundation (NSF) encourages the principal investigator (PI) to conduct an experiment to help provide evidence of a causal link between culturally responsive teaching (CRT) and students' mathematics achievement. During the previous two years of the project, the professional development team developed a summer program to train teachers to use CRT practices in high school classrooms in urban schools. Additionally, the research team developed a reliable and valid observation protocol for CRT. Although the project works with secondary teachers who teach in a variety of school subjects, the research team uses a standardized mathematics test for secondary school with success.

In January 2011, the PI admitted 25 secondary teachers who teach in urban schools as the current cohort (the third in the project). The teachers have a variety of backgrounds and experience, about half (13 teachers) having over 10 years of teaching experience. The PI selected all the teachers based on their eagerness to explore new teaching techniques and expand their practices.

Describe the challenges associated with conducting educational research to address such questions and discuss various approaches in the literature to address these challenges. Propose a study to the PI based on weighing the challenges that you have discussed.

3. There is significant existing research on students' understanding of limits of sequences and series, definite integrals and infinite sets in the mathematics education literature. One area that has received little attention, however, is the role of indexing and index notation. Propose a study to reveal how cognitive issues related to indices affect students' thinking and learning of other concepts in which they are used. Outline an appropriate theoretical perspective to frame your study and justify your choice. Pose specific research questions in terms of this perspective and articulate their connection to existing research on limits and infinite sets. Discuss how relevant findings from the literature on function concepts inform your design and expectations.

## MED COMPS

Summer 2011

### Part II

**ANSWER TWO** (3 hours with computer). Respond to TWO of the three items. This part of the exam focuses on the reading-list articles.

Provide responses to entire questions. All responses should be in essay form; aim for clarity and explicitness, as well as thoroughness, concision, and coherence in your writing. Define all technical terminology that you use in your responses and be explicit about how you are applying the ideas.

- Start each answer on a new sheet of paper and double space the final version. If you use diagrams in your responses, label each with a title (e.g., "Figure x") and insert a clear reference to each one in the appropriate place in your narrative.
  - In all cases, support assertions with citations from the literature, as appropriate, using APA format: (Author(s), Year).
  - Print out a copy of your response AND send an electronic copy by email to [Hortensia.soto@unco.edu](mailto:Hortensia.soto@unco.edu).
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1. The state of Colorado Department of Education (CDE) created a new masters program where inservice elementary teachers would become elementary mathematics specialists (EMS). They would become teacher leaders with specialized knowledge and be responsible for supporting effective PK-6 mathematics instruction and student learning. These specialists will fill a variety of roles, including:
  - teaching mathematics to elementary students in their regular mathematics class,
  - providing intervention and enrichment instruction to small groups and/or individuals,
  - supporting other elementary teachers in effective mathematics instruction through coaching and professional development, and
  - working with school and district leaders to strengthen mathematics instructional programs.

The CDE has hired you to investigate the effectiveness on student learning from having such specialists in the Colorado schools. Describe how your might conduct mixed-methods research using the methods employed by Speer and Wagner, (2009), Clements, Sarama, Spitler, Lange, and Wolfe (2011) and Ross, McDougall, Hogaboam-Gray, and LeSage (2003). Be sure to discuss how you would incorporate

HLM methods and how you could ensure the various validities discussed by Ross et al. (2003).

2. Suppose Gresalfi and Cobb (2011) used the motivational framework described by Dweck and Leggett (1988) in their research. How might their data collection, analysis, results and conclusions be different? How might they be the same? Be sure to support your reasoning.
3. Gravemeijer, Cobb, Bowers, & Whitenack (2000) advance a design perspective on educational research called “Realistic Mathematics Education” (RME) that falls within a broad category of Social Constructivism. Apply Salomon & Perkins’ (1998) delineation of different perspectives on social learning to elaborate the details of RME. In particular, discuss the meaning and implications of characterizing the “model-of/model-for transition” in terms of “activity.” Elaborate the social characterization of a “hypothetical learning trajectory,” the guiding heuristics, and the units of analysis within RME and discuss how these elements are classified in Salomon & Perkins’ terms.

## MED Comps: Summer 2010

### PART I

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  - In all cases, support assertions with citations from the literature, as appropriate, using APA format: (Author(s), Year).
  - Print out a copy of your response AND send an electronic copy by email to [bill.blubaugh@unco.edu](mailto:bill.blubaugh@unco.edu).
1. The Colorado Council of Teachers of Mathematics (CCTM) in cooperation with the Colorado Department of Education (CDE) will present a series of regional workshops designed to support teachers in building readiness for implementing Colorado's revised mathematics standards. They will present the workshops between the dates of August 28<sup>th</sup> and September 25<sup>th</sup> in all seven CCTM regions and some regions will have duplicate workshops in order to accommodate the teachers from these bigger regions. The table below indicates the regions, dates, and locations of the workshops.

Region 1	September 11	Littleton
Region 2	August 28	Lakewood/Denver
Region 3	September 11	Bennett
Region 4	August 28	Fort Collins
Region 4	September 18	Fort Morgan
Region 5	September 11	Alamosa
Region 5	September 25	Pueblo
Region 6	August 28	Grand Junction
Region 6	September 18	Montrose
Region 7	August 28	Colorado Springs

As a result of attending the workshop, participants will be able to answer these key questions:

- ❖ How will 21<sup>st</sup> century skills support our students' readiness for post-secondary opportunities?
- ❖ How will 21<sup>st</sup> century skills look and sound in our mathematics classrooms?
- ❖ What are the characteristics of tasks that will support the development of 21<sup>st</sup> century skills?

- ❖ What are teacher moves that support students in developing 21<sup>st</sup> century skills?

You have been asked to serve as a consultant to evaluate the effectiveness of this workshop. In particular CCTM and CDE would like you to conduct a mixed methods study that addresses the following questions:

- ❖ What is the nature and magnitude of differences, if any, in teachers' ability to answer the above questions based on whether they participated in the workshop?
- ❖ What is the nature and magnitude of differences, if any in teachers' competence at implementing Colorado's revised mathematics standards?

Your task is to present CCTM and CDE with a proposal of how you might conduct this research. Be sure to include the following:

- pertinent research that informs your research design along with an explanation of how it informs your research design,
  - summary of your research design and materials,
  - list of research variables (independent and dependent) with a brief rationale for each variable,
  - summary of data collected and data collection methods,
  - discussion of factors that might need to be considered in the study,  
  
including issues of reliability and validity.
  - summary of methods and justification for the analysis of such data, including the unit of analysis and the descriptive and inferential statistical methods and tests that would be appropriate, and a
  - discussion of the issues of internal and external validity that might arise from the proposed design
2. Researchers have documented the negative relation between anxiety and mathematics achievement. For example, in a meta-analysis, Ma (1999) formed three grade-level groups: 4–6, 7–9, and 10–12. As indicated in the abstract below, the relationship between anxiety and mathematics achievement was consistently negative across grade levels. Although this relation is generally negative, how anxiety impacts individual students' mathematics achievement is of interest to many educators. Design a qualitative study that addresses the nuanced interaction of anxiety and mathematics achievement.
- From your knowledge of the literature, determine at least three articles on which to base your study. Briefly explain why each article is valuable and appropriate.
  - Carefully detail and justify your design for the study.

**Abstract:**

In this meta-analysis I examined 26 studies on the relationship between anxiety toward mathematics and achievement in mathematics among elementary and secondary students. The common population correlation for the relationship is significant ( $-.27$ ). A series of general linear models indicated that the relationship is consistent across gender groups, grade-level groups, ethnic groups, instruments measuring anxiety, and years of publication. The relationship, however, differs significantly among instruments measuring achievement as well as among types of publication. Researchers using

standardized achievement tests tend to report a relationship of significantly smaller magnitude than researchers using mathematics teachers' grades and researcher-made achievement tests. Published studies tend to indicate a significantly smaller magnitude of the relationship than unpublished studies. There are no significant interaction effects among key variables such as gender, grade, and ethnicity.

#### References:

Ma, X. (1999). A meta-analysis of the relationship between anxiety towards mathematics and achievement in mathematics. *Journal for Research in Mathematics Education*, 30, 520-540.

Tall, D., McGowen, M, & DeMarois, P. (2000, October). The function machine as a cognitive root for the function concept. Paper presented at the Twenty-second Annual Meeting of the International Group for the Psychology of Mathematics Education, Tucson, AZ.

3. Mathematics education research findings are organized and linked to theories because they serve as an overarching framework to which the data and results could be linked. Two learning theories that are on opposite sides of the spectrum are behaviorial theories and constructivist theories. Select a theory that falls under each of these overarching branches and compare and contrast how followers of these theories would answer each of the following questions.
  - a. How does learning occur?
  - b. What factors influence learning?
  - c. What is the role of memory?
  - d. What is the role of motivation?
  - e. How does transfer occur?
  - f. What processes are involved in self-regulation?
  - g. What are the implications for teaching and assessment?
  - h. What are the implications for research?



## MED COMPS

### Summer 2010

#### Part II

**ANSWER TWO** (3 hours with computer). Respond to TWO of the three items. This part of the exam focuses on the reading-list articles. Each answer is a complete essay.

- Start each answer on a new sheet of paper and double space the version you turn in.
  - In all cases, support assertions with citations from the literature, *within and beyond* the group of seven readings accompanying this part of the exam, using APA format: (Author(s), Year).
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1. It was recently documented “that over the past five decades, the number of hours that the average college student studies each week has been steadily dropping. According to Babcock and Marks (2010), “the average student at a four-year college in 1961 studied about 24 hours a week. Today’s average student hits the books for just 14 hours.” Their findings indicated that this phenomenon was not dependent on major, gender, race, the size of the school, or SAT scores. Although some attribute these results to computers, Babcock and Marks found the greatest decline in student studying between 1961 and 1981; “study times fell from 24.4 to 16.8 hours per week (and then, ultimately, to 14).” You would like to investigate these results further in order to try to explain these results. Describe and explain how you might connect and incorporate the theoretical perspectives, research designs, analyses, and results used by Post et al. (2010) and Cobb (1994) in order to:
    - a. derive a research question that would require mixed-methods,
    - b. determine data collection methods and analysis, and
    - c. to define a theoretical perspective.
  2. Hill, Ball, and Schilling (2008) propose a domain map for subject matter knowledge and pedagogical content knowledge for teaching mathematics. Leonard, Brooks, Barnes-Johnson, and Berry (2010) examine the “nuances and complexities” of teaching mathematics using culturally relevant instruction that promotes social justice. Compare and contrast the kinds of knowledge needed to teach mathematics identified by Hill, Ball, and Schilling (2008) with the knowledge needed to teach mathematics for cultural relevance and social justice (Leonard et al., 2010).

3. In a high school geometry class, Jayden makes the claim that “Two convex quadrilaterals are congruent if they satisfy the SASA proposition i.e. two convex quadrilaterals  $ABCD$  and  $EFGH$  are congruent if  $AB \cong EF$ ,  $BC \cong FG$ ,  $\angle B \cong \angle F$ , &  $\angle C \cong \angle G$ .”

Jayden provides the following proof.

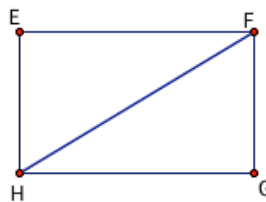
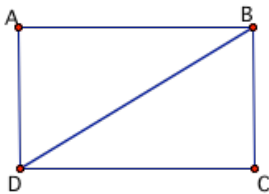
Pf: Let quadrilaterals  $ABCD$  and  $EFGH$  be rectangles, which are obviously convex since none of the angles are greater than 90 degrees and such that  $AB \cong EF$ ,  $BC \cong FG$ ,  $\angle B \cong \angle F$ , &  $\angle C \cong \angle G$ .

Construct segment  $BD$ , which bisects angle  $B$  and  $D \rightarrow \angle ABD \cong \angle CBD$  &  $\angle BDC \cong \angle BDA$ .

Similarly, if we construct segment  $FH$ , it will bisect angles  $F$  and  $H \rightarrow$

$\angle EFH \cong \angle GFH$  &  $\angle GHF \cong \angle EHF$ .

Since angles  $B$  and  $F$  are right angles and we bisected them we have the following equality:  $\angle DBC \cong \angle HFG$ . Thus by ASA we have that triangle  $BCD$  is congruent to triangle  $FGH$ .



Also by SSS we have that triangle  $ABD$  is congruent to triangle  $CDB$  and triangle  $EFH$  is congruent to triangle  $GHF$ . Thus, by the transitive property we have that triangle  $ABD$  is congruent to triangle  $EFH$ .

Since all the corresponding sides and angles of the two quadrilaterals are congruent then the two quadrilaterals are congruent. Thus, SASA holds for convex quadrilaterals.

Since all the corresponding sides and angles of the two quadrilaterals are congruent then the two quadrilaterals are congruent. Thus, SASA holds for convex quadrilaterals.

Compare and contrast how Inglis & Mejia-Ramos (2008) and Lobato & Siebert (2002) would interpret Jayden's conjecture and proof. What factors might they provide that would explain Jayden's conjecture and proof? What suggestions might these researchers provide for Jayden's teacher that might help alleviate Jayden's misconceptions? How would these suggestions compare and contrast to suggestions from Hill, Ball, & Schilling (2008) and Speer & Wagner (2009)?

## MED Comps: Summer 2009

### PART I

**ANSWER ALL** (3 hours with computer). This part of the exam focuses on general knowledge, mainly from coursework and readings in MED 610, 700, 701 and 702.

- Start each answer on a new sheet of paper and double space the version you turn in.
  - In all cases, support assertions with citations from the literature, as appropriate, using APA format: (Author(s), Year).
  - Print out a copy of your response AND send an electronic copy by email to hortensia.soto@unco.edu.
2. Consider the following abstract by Chapman (1997) for the article “Metaphors in the Teaching of Mathematical Problem Solving”.

*This article reports on a study that investigated the teaching of mathematical problem solving from a teacher's perspective. The study focused on three teachers and their way of making sense of teaching problem solving. Data collected through interviews and classroom observations were analyzed in the context of an interpretive qualitative study to understand the meanings of the participants' classroom processes. The findings indicated that the participants unconsciously constructed personal metaphors that became the basis of their conceptualization of problems and making sense of their teaching. “Community”, “adventure” and “game” were determined to be the key metaphors of the three participants respectively. These metaphors embodied their personal experiences and personal practical beliefs that provided the unique meaning associated with their classroom processes. The outcome suggested that the study of such metaphors could be a promising avenue in enhancing teacher education and in problem solving research in the quest to make the teaching of problem solving more meaningful and effective in a classroom context.*

You would like to investigate the author's claim that *the study of such metaphors could be a promising avenue in enhancing teacher education* through a quantitative lens.

- a. Write a research question that could extend this research through a quantitative perspective.
- b. Describe pertinent research that would inform your research design and explain how it would inform your research design.
- c. Summarize your research design and instruments.
- d. Identify the research variables (independent and dependent) and provide a brief rationale for each choice.
- e. What data might be collected and how might it be collected? Discuss factors that might need to be considered in the study, including issues of reliability and validity.
- f. Discuss methods and justification for the analysis of such data, including the unit of analysis and the descriptive and inferential statistical methods and tests that would be appropriate.

- g. Discuss the issues of internal and external validity that might arise from the proposed design.
3. Identify and describe research on the teaching and learning of mathematics from the perspective of 3 different cognitive theories. Compare these theories in cognition and provide the rationale as to the conditions under which you might support one theory over the others.
  4. You have been asked to review proposals for presentation at an international research conference. Carefully outline and analyze *at least two* strengths and *at least two* weaknesses of the *design* of the qualitative study summarized below.

**Background.** In preparing elementary teachers to teach mathematics in urban schools, a tension exists between facilitating candidates' development of content and pedagogical content knowledge and helping candidates to be advocates for social justice in their schools and communities. Oakes, Franke, Quartz, & Rogers (2002) highlight this tension in their discussion of "high-quality urban teaching" (p. 229). At the same time, those who are preparing to teach in high-poverty schools are often confronted by the lack of material resources - including curriculum materials, manipulatives, and technology - available for teaching mathematics, raising significant questions about equity and about the effectiveness of university-taught methods classes based on the use and availability of these resources. The purpose of this study is to generate a theoretical framework for an elementary mathematics methods course that seeks to address not only issues of equity, but also the tensions among content knowledge, pedagogical content knowledge, and a commitment to social justice. This framework builds on the work of Oakes et al. (2002) as well as a number of currently disconnected bodies of research highlighting the *human* and *social* resources available for the teaching of mathematics. In recent years, researchers have provided evidence that helping in-service teachers access and build on the mathematical "funds of knowledge" of students' families and communities can be a powerful mechanism for supporting the mathematical learning of students, parents, and teachers (e.g., Civil, Andrade, & Anhalt, 2000; Civil & Bernier, 2006; Gonzalez, Moll, & Amanti, 2005). Others have proposed "third space" as a theory for understanding the learning and exchange of capital that can occur with individuals from diverse communities and practices come together for the purpose of teaching and learning (e.g., Moje et al., 2004). While all of these theories hold great promise and have led to significant (though typically small scale) successes with *in-service* teachers, very little research exists documenting or theorizing how teacher educators might create the kinds of spaces needed for *pre-service* teachers to learn to access and build on funds of knowledge, take advantage of diversity as a resource, and build productive third spaces in their own classrooms (though Gonzalez, Moll & Amanti, 2005, and Rodriguez & Kitchen, 2005, have each begun to describe this process). Here, we build on the theoretical constructs of funds of knowledge and third space to develop a framework for understanding the ways in which pre-service teachers learn about families and communities as resources for instruction for both literacy and mathematics.

**Methods.** Two cohorts (N=51) of elementary literacy and mathematics methods students have participated in a new partnership between the university and a local elementary school. This partnership is designed to help pre-service teachers learn about incorporating family and community resources into literacy and mathematics instruction. The local elementary school provides an interesting context for this work because its student body has undergone significant transitions in recent years, due to school closures, redrawn district boundaries, and increasing rates of immigration and poverty in the local community. For example, the percentage of students receiving free and reduced lunch has more than tripled in the past three years.

Data for the project include: 1) Interviews with pre-service teachers at the beginning and end of the semester; 2) Interviews with in-service teachers and school principal at the end of each semester; 3) Pre-

service teachers' work, including lesson plans, reflective essays, and blogs; and 4) Field notes from Family Math and Literacy Nights (organized and run by pre-service teachers). These data were analyzed using open and iterative coding in which codes were initially assigned based on the research questions and then refined as findings emerged from the data. In this paper, we focus in particular on the pre-service teachers' interviews and reflections from the second semester of the project to address the research question: What are the boundaries, key activities, tools, roles, and resources that define the overlaps and third spaces among the communities and funds of knowledge involved in an elementary school-university methods collaboration?

## MED COMPS

Summer 2009

### Part II

**ANSWER TWO** (3 hours with computer). Respond to TWO of the four items. This part of the exam focuses on the reading-list articles. Each answer is a complete essay.

- Start each answer on a new sheet of paper and double space the version you turn in.
  - In all cases, support assertions with citations from the literature, *within and beyond* the group of seven readings accompanying this part of the exam, using APA format: (Author(s), Year).
  - Print out a copy of your response AND send an electronic copy by email to [hortensia.soto@unco.edu](mailto:hortensia.soto@unco.edu).
1. Use the readings for this part of the exam and other literature with which you are familiar to illustrate your argument in answering the following multi-part question: What implications does constructivism have for grades 7-12 student learning, for grades 7-12 classroom instruction, and for grades 7-12 teacher education? In what way(s) is this different from what might be true at the college level? Why is it different? Be sure to include in your discussion substantive connections to the readings for the exam as well as to references in the history and philosophy of mathematics education and to recent work on student conceptions.
  2. Describe the implications that Lobato & Siebert (2002) research on transfer has for research on unpacking pedagogical content knowledge as described by Hill, Ball & Schilling (2008). How might Hill and her colleagues use Lobato & Siebert's work to extend their own research?
  3. You are interested in designing a mixed methods study that investigates how undergraduate preservice secondary mathematics majors learn and understand probability. Describe and explain how you might connect and incorporate the theoretical perspectives, research designs, analyses, and results used by Hill, Ball & Schilling (2008), Harwell et al. (2007), Pirie & Kieren (1994), and Dalhberg & Housman (1997) in order to:
    - d. derive a research question that would require mixed-methods,
    - e. determine data collection methods and analysis, and
    - f. to define a theoretical perspective.
  - a. Describe how Weber's (2008) work complements Herzig's (2002) work and vice-versa. How might they each use the other's research to extend their own research?

**Part I – ANSWER ALL** (3 hours with computer). This part of the exam focuses on general knowledge, mainly from coursework and readings in MED 610, 700, 701 and 702.

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1. A pair of researchers is designing a mixed methods study of mathematics cognition among the Associate of Arts (AA) degree students in music at a Performing Arts Community College (PACC) in a major metropolitan area. The research question is:

*What is the nature of College Algebra learners' cognition in generating representations of linear relationships?*

In the music AA program at PACC, student demographic information includes the following: 48% of the students are European American, 34% of the students are Mexican American, 11% of the students are African American, 2% of the students are Filipino American, and the rest of the students are of mixed ethnic/racial background (the demographics of the mathematics faculty at PACC is about the same). Each AA student in music attends five courses on a block schedule with one of the courses being a mathematics course (e.g., two 75 minute classes in mathematics each week, on Monday/Wednesday or Tuesday/Thursday).

From your knowledge of the literature, particularly on the uses of learning and cognition theories in research about similar topics, what research reports (*give at least 3*) do you recommend the researchers read in preparing to design their study? Explain why each article is valuable and appropriate to the PACC study.

2. You are planning to conduct a quantitative research study in which you address the question: *Does mathematics education research impact the teaching of mathematics graduate teaching assistants (GTAs)?* In one essay address all of the following (it is not necessary to address them in the order given).

- a. What pertinent research do you think graduate teaching assistants should be exposed to and how would you use this to create your research design and research materials?
  - b. Identify the research variables (independent and dependent) and provide a brief rationale for each choice.
  - c. What data might be collected and how might it be collected? Discuss factors that might need to be considered in the study, including issues of reliability and validity.
  - d. *Describe* and *justify* methods for the analysis of such data, including the unit of analysis as well as the descriptive and inferential statistical methods and tests that would be appropriate.
  - e. Discuss issues of internal and external validity that might arise from the proposed design. How might you address them?
3. You have been asked to review proposals for presentation at an international research conference. Carefully describe and analyze *at least two strengths* and *at least two weaknesses* of the *design* of the qualitative study summarized below.

A number of mathematics researchers are calling for teachers to both teach students mathematics and facilitate students in using mathematical knowledge to identify and confront obstacles to their success (Gutstein, 2003; Martin, 2003; Secada, 1989). Teaching mathematics for social justice addresses both aspects of this call. Despite the potential teaching mathematics for social justice has in addressing issues of inequity in mathematics education (Skovsmose, 1994; Gutstein, 2003), little research exists that examines *mathematics teachers learning to teach for social justice*. The research study I conducted represents a first attempt to address this gap in the literature. This study investigated secondary mathematics teachers' conversations around learning to teach mathematics for social justice as they participated in a graduate course that engaged them in a version of "lesson study" (Stigler & Hiebert, 1999). Teachers developed, implemented, observed, revised and re-taught mathematics lessons incorporating social justice goals. Specifically, the study was guided by two research questions: (1) How do teachers' conversations around teaching mathematics for social justice evolve through participation in the graduate course? (2) What challenges do teachers recognize they face in teaching mathematics for social justice?

To address these research questions, I designed a graduate course to provide secondary mathematics teachers an opportunity to explore their conceptions of teaching mathematics for social justice through assigned readings and written reflections, and through the collective development of a mathematics lesson incorporating social justice goals. Eight teachers signed up to participate in the graduate course, which met 15 times for 2 1/2 hours per session. Of these eight teachers, seven were employed as mathematics teachers in one of four comprehensive high schools and the eighth was a licensed science teacher employed as a full-time substitute teacher, regularly substituting in mathematics classrooms. For scheduling purposes, these teachers were split into two groups of four. In Group 1, all four teachers were White, two were female and two were male, and their teaching experience ranged from 4 to 17 years. Three of these teachers taught in the same high school and the fourth was the substitute



teacher. In Group 2, all of the teachers were White and female, and their teaching experience ranged from 6 to 16 years. The teachers in this group taught at three different high schools. The central activity for the first part of the course was discussion and analysis of readings focused on teaching for social justice in general and teaching mathematics for social justice specifically. To situate their study of these readings, the group also examined sample mathematics lessons and mathematics teaching cases. I provided verbal and written reflection prompts in each session. The central activity in the second part of the course was engagement in lesson study: teachers designed, implemented, observed, revised, and re-taught a mathematics lesson for social justice. (For example, teachers in Group 1 developed a lesson around institutional racism as “the” explanation for racial disparities in student GPA values and teachers in Group 2 developed a lesson examining differences between minimum and living wages.)

The primary source of data for this study was teachers’ discussions during the course. All discussions were audio-taped and transcribed. Additionally, pre- and post-seminar interviews were conducted with all participants to help understand how teachers’ conversations around teaching mathematics for social justice evolved. Teachers’ lesson plans and written reflections were also collected. A grounded theory methodology was employed (Strauss & Corbin, 1990) to identify recurring themes in the data. I began first by compiling a list of general framing codes drawn from my research questions. Next, I coded all interview transcripts, seminar session transcripts, written reflections and written lesson plans via “open coding” within these framing codes. The emergence of additional codes occurred through multiple passes of the entire data set; four passes were required before categories began to stabilize. The coding scheme aimed to characterize the nature and content of teachers’ comments.

**PART II. – ANSWER TWO** (3 hours with computer). Respond to TWO of the four items. This part of the exam focuses on the reading-list articles.<sup>1</sup> Each answer is a complete essay.

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1. Compare and contrast the need for everyday skills and abstract skills in mathematics as described by Carraher, Carraher & Schliemann (1985) and Harwell et al. (2007).

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<sup>1</sup>MED Comp 2008 Reading List:

Balfanz, R., & Byrnes, V. (2006). Closing the mathematics achievement gap in high-poverty middle schools: Enablers and constraints. *Journal of Education for Students Placed at Risk, 11*(2), 143-159.

Boaler, J. (1998). Open and closed mathematics: Student experiences and understandings. *Journal for Research in Mathematics Education, 29*(1), 41-62.

Carraher, T. N., Carraher, E. W., & Schliemann, A. D. (1985). Mathematics in the streets and in schools. In T. Carpenter, J. Dossey, & J. Koehler (Eds.), *Classics in mathematics education research* (pp. 187-193). Reston, VA: National Council of Teachers of Mathematics.

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2. Compare and contrast the *theoretical perspective(s)*, *research design*, and *methods of reporting* in Balfanz and Byrnes (2006) and Boaler (1998).
3. Compare and contrast the *epistemological perspective(s)*, *researcher perspective towards the researched*, and *methods of reporting* in Tall and Vinner (1981) and Erlwanger (1973).
4. Using any criteria for evaluating quantitative research with which you are familiar (e.g., Gall, Gall, & Borg, 2003; Schoenfeld, 2000), evaluate the report by Kaminski, Sloutsky, and Heckler (2008). Independent of which criteria you use, discuss: (a) the basis for the research questions, (b) sampling and population, (c) experimental design, (d) data collection process, (e) results, and (f) implications of the study.

## Mathematics Education Comprehensive Exams

Summer 2007

Part I (3 hours with computer): This part of the exam focuses on general knowledge, mainly from coursework and readings in MED 610, 700, 701, and 702. Start each answer on a new sheet of paper and double-space the text. **Answer ALL 3 questions.** In all cases, support assertions with citations from the literature, as appropriate, using APA format: (Author(s), Year). Print out a copy of your responses AND provide an electronic copy (either e-mail a copy to [bill.blubaugh@unco.edu](mailto:bill.blubaugh@unco.edu) or save to a memory stick or floppy and give the stick or floppy to the proctor to save an electronic copy).

2. Suppose that you were going to conduct a study of middle school students' developing understandings of linearity across a Linear Functions unit in the mathematics curriculum. Select, describe, and defend an appropriate learning theory that would serve as the theoretical framework for your study. Then, describe how the learning theory would influence your study, i.e., how it might influence the educational intervention, variables investigated, research design, data collection, data analysis, and conclusions.
3. You would like to conduct a quantitative research study in which you address the question: Does mathematics education research impact secondary mathematics teacher's teaching?
  - a. What pertinent research do you believe secondary mathematics teachers should be exposed to and how would you use this to create your research design and materials?
  - b. Identify the research variables (independent and dependent) and provide a brief rationale for each choice.
  - c. What data might be collected and how might it be collected? Discuss factors that might need to be considered in the study, including issues of reliability and validity.
  - d. Discuss methods and justification for the analysis of such data, including the unit of analysis and the descriptive and inferential statistical methods and tests that would be appropriate.
  - e. Discuss issues of internal and external validity that might arise from the proposed design.
4. Two topics for possible research studies have been given below. CHOOSE ONE and explain how you would design a qualitative research study to examine the topic. Be sure to describe your research question/purpose, theoretical framework, data collection, and data analysis procedures.

- a. Knuth et al. (2006) provide the following abstract for their quantitative research paper, *Does Understanding the Equal Sign Matter? Evidence from Solving Equations*.

*Given its important role in mathematics as well as its role as a gatekeeper to future educational and employment opportunities, algebra has become a focal point of both reform and research efforts in mathematics education. Understanding and using algebra is dependent on understanding a number of fundamental concepts, one of which is the concept of equality. This article focuses on middle school students' [N=177] understanding of the equal sign and its relation to performance on solving algebraic equations. The data indicate that many students lack a sophisticated understanding of the equal sign and that their understanding of the equal sign is associated with performance on equation-solving items. Moreover, the latter findings hold even when controlling for mathematics ability (as measured by standardized achievement test scores). Implications for instruction and curricular design are discussed.*

You would like to follow-up on this study in order to investigate why students lack a sophisticated understanding of the equal sign. More specifically, you would like to describe how middle school students understand the equal sign.

- b. In Gutstein, Lipman, Hernandez, and de los Reyes (1997), they suggest follow-up research to investigate how someone from outside the culture becomes a culturally relevant teacher. You want to investigate this process.

Part II (3 hours with computer): This part of the exam focuses on applications of knowledge in the context of the eight assigned readings. Respond to **TWO** of the four questions. Each answer should be a complete essay. Start each one on a new page and double-space the text. Support your assertions with citations from key literature (within and beyond the group of eight readings<sup>2</sup> accompanying this part of the exam) using APA format: (Author(s), Year). Print out a copy of your responses AND provide an electronic copy (either e-mail a copy to [bill.blubaugh@unco.edu](mailto:bill.blubaugh@unco.edu) or save to a memory stick or floppy and give the stick or floppy to the proctor to save an electronic copy).

1. Compare and contrast a conception-based perspective with a perception-based perspective as described in Simon et al. (2000). Compare and contrast these two perspectives with the understandings advocated in Cobb et al. (2001). Finally, describe how teachers' perspectives evolving from a perception-based perspective to a conception-based perspective help teacher educators and/or mathematics students.

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<sup>2</sup> Cobb, P., Stephan, M., McClain, K., & Gravemeijer, K. (2001). Participating in classroom mathematical practices. *The Journal of the Learning Sciences*, 10(1 & 2), 113-163.

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Simon, M. A., Tzur, R., Heinz, K., & Kinzel, M. (2000). Characterizing a perspective underlying the practices of mathematics teachers in transition. *Journal for Research in Mathematics Education*, 31(5), 579-601.

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2. Compare and contrast the statistical analysis of Healy and Hoyles (2000) and Harwell et al. (2007). Describe the strengths and weaknesses of the design of study and discuss the appropriateness of the statistical analysis conducted. How could the research design be improved for each study?
3. Compare and contrast the opportunities for participating in a mathematical classroom community by the elementary/middle school students described in Gutstein et al. (1997) and by the doctoral students described in Herzig (2002).
4. Describe the concept image (Tall & Vinner, 1981) of the students described in Healy and Hoyles (2000) with regard to writing, evaluating, and the utility of proofs.