TEACHING INQUIRY TO HIGH SCHOOL TEACHERS THROUGH THE USE OF MATHEMATICS ACTION RESEARCH PROJECTS

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Abstract: This paper describes the use of mathematics Action Research Projects (ARPs) as a capstone experience in lieu of a Master's thesis in a Master's program for in-service secondary teachers. The ARPs include two primary components: 1) each teacher participant conducts mathematical research that is new to them on a topic that is connected in some way to the material they teach in one of their classes; and 2) each teacher participant also creates an in-class innovation related to the mathematical research they have been doing, and implements it in their own classroom. Ten teacher participants have completed such ARPs with the author over the last three years. In completing their ARPs, teacher participants not only learn how to conduct mathematical inquiry themselves, but also most often choose to implement some kind of inquiry-based classroom innovation that allows them to share some of this experience with their own students. This course structure has had the effect of encouraging teachers to decide to try more inquiry-based teaching methods, and at the same time, has given them the knowledge and tools to be successful in doing so.

Keywords: teaching inquiry, high school teachers, action research

1 INTRODUCTION

The University of Northern Colorado has a long-established Master's program for in-service secondary teachers. To avoid confusion, the teachers enrolled in this program will be referred to as "participants" throughout this paper, since they are simultaneously students in our program and teachers in their own classrooms. The program combines online courses during the academic year with a mixture of online and face-to-face classes during the summer. This program has always had a capstone requirement in lieu of a formal Master's Thesis. Originally, this capstone requirement was the development of a curriculum module that might or might not have been implemented in a participant's own classroom. Then, about ten years ago, the capstone requirement was changed to a Mathematics Education Action Research Project (ARP), in which participants modify and study some aspect of their own teaching in their own classroom, using mathematics education research methods.

Action Research Projects are a commonly used research methodology in mathematics education settings. See, for example, [7], [8], and [13]. According to the American Educational Research Association (AERA) Action Research Special Interest Group, "Action research seeks transformative change through the simultaneous process of taking action and doing research, which are linked together by critical reflection. Action research practitioners reflect upon the consequences of their own questions, beliefs, assumptions, and practices with the goal of understanding, developing, and improving social practices. ...Critical reflection on action and reflexive writing are key and central processes of action research." [15] Action Research Projects have a long history of use with in-service teachers, in particular, as a way to empower the teachers to be more reflective about their own teaching and engage in a cycle of continuous inquiry, self-evaluation and improvement. [2, 3, 4] Participants in our program are supported in conducting their ARPs through a three credit course taught by a faculty mentor, who also serves as the chair of a two-person Master's committee that evaluates the completed project.

Five years ago, it was recognized that some of the participants would be more interested in a capstone that focused more on mathematics than on mathematics education research, either because they preferred doing mathematical research to doing educational research, or else because they wanted additional mathematics credits. (Many teachers need 18 graduate credit hours in mathematics in order to be qualified to teach dual-credit courses in their high schools.) As a result, we created an option for a "Mathematics Action Research Project." In completing these Math ARPs, participants had the opportunity to do research on a mathematical subject. For some participants this included mathematical inquiry that they conducted themselves on a topic new to them (although other people might have previously looked at the same or similar topics), while for other participants, this research primarily involved reading about and trying to understand what other mathematicians had done on a given topic. Participants were also supposed to include a section relating this research in some way to their own teaching.

Three years ago, I set out to revamp the Mathematics Action Research Projects, trying to incorporate what I thought were the best aspects of all of these previous models, and including a strong inquiry focus. This paper describes the resulting revamped Mathematics Action Research Projects, which have been successful not only at getting all of the participants to conduct original (to them) mathematical research, but have also led many of them to try out inquiry-based teaching in their own classrooms.

2 MATHEMATICS ACTION RESEARCH PROJECTS

The revamped mathematics Action Research Projects have two principal requirements:

- Each participant must conduct original (to them) research on a mathematical topic that is connected in some way to one of the classes they are teaching; and
- Each participant must develop and implement an innovation in their own classroom that is related in some way to the mathematical research they have been doing, and reflect on how it went.

Participants typically spend one semester working on their project. At the end of the semester, they turn in a final written draft of their project, and make an oral presentation about it to their two-person committee and their classmates.

Participants are given wide latitude to design an innovation to try in their classrooms. They are asked to use the extended period of time they have while working on their ARP to develop an innovation that will push their teaching in a direction that they want it to go, but that they otherwise might not have the time and energy to try out. The connection between the teaching innovation and the participant's mathematical research could be one of content (the participant's students study a topic related to something that the participant studied) or of style (the participant's students study something in way similar to the way the participant studied their research topic; for example, by using the same manipulatives or software).

The intent of these new math ARPs is to combine some of the best aspects of all of the previous versions of the capstone requirement. I have always felt that the best aspect of most Mathematics Education ARPs is the development of a classroom innovation, because it gives teachers the opportunity to carefully think about some aspect of their teaching that they want to change, and to spend a significant amount of time thinking about how to best implement it. They also try to carefully study their innovations using tools from mathematics education, but this part of the ARP is often less successful, because most participants, who are Master's students, don't really have the training, time, or background to design a study that will give meaningful results beyond their own reflections on how it went. The curriculum modules had the advantage of getting participants to think about what would be possible to do in their classrooms, and were connected to particular mathematical content in a way that the MED ARPs are not, but they weren't required to be actually implemented in the classroom, and they didn't involve any original research. Finally, the previous math ARPs were connected to a mathematical content topic, but it wasn't usually one directly related to the content that the teachers were teaching, and the connections that participants made to their own classrooms were often tenuous. Furthermore, the research that participants did tended to focus more on understanding what other people had done than on conducting their own mathematical inquiry.

In the revamped math ARPs, participants do their own mathematical research on topics related to the material that they are actually teaching. They start out by doing their own inquiry into these topics, without looking at what other people have previously done first, and then later connect what they have done to other people's work. They also design and implement a classroom innovation that is related to their mathematical research topic, and reflect on how it went. This has a number of advantages:

- The participants get to have an extended experience doing their own mathematical inquiry, at whatever level is appropriate for them, into topics that are meaningful to them.
- They get to design a classroom innovation carefully, without having to worry about how to formally measure how effective it is.
- Because they have just experienced the joy and feeling of accomplishment that comes from making their own mathematical discoveries, they often want to share this with their students. Furthermore, they have just become expert in the materials they are researching, and therefore feel comfortable guiding their students through inquiries in the same area. So, although the class structure doesn't require the classroom innovation to be inquiry-based in any way, in actual practice, most participants end up developing inquiry-based lesson plans that mirror some of their own work.

Thus, this course structure has the effect of encouraging teachers to decide to try more inquiry-based teaching methods, and at the same time, gives them the knowledge and tools to be successful in doing so.

3 STRUCTURE OF THE CLASS

All of the participants who are working on an ARP at the same time enroll in a 3 credit class taught by a faculty mentor (in this case, me). This is a way of giving the participants credit for the work they are doing, giving the faculty advisor credit for mentoring the projects, and providing structure for what would otherwise be an unmanageably large project. Because the participants in the class are in-service teachers, the class meets online. We meet synchronously online using BlackBoard Collaborate videoconferencing classroom software for one hour once a month. During these meetings, the participants compare notes about how their projects are going, explain their research topics, and seek advice from one another about their classroom innovations. They are also strongly encouraged to check in with me about their projects outside of our class meetings at least once a week and whenever they are feeling stuck.

Participants are required to submit 6 written drafts of their ARP throughout the semester, most of which receive detailed written feedback:

- A written proposal that briefly outlines the participant's proposed area of mathematical research and proposed innovation. This proposal is typically due during week 3 of the class.
- Draft A: First draft of the section of the paper describing the participant's mathematical problem and their initial work on this problem. At this point, participants are not allowed to consult any outside sources about their mathematical problem unless I have specifically suggested them. I let participants know on an individual basis when I think they are far enough along on their problem to look at outside sources. This draft is due during week 6 of the class.
- Draft B: Second draft of the paper incorporating further work on the mathematical problem, revisions in response to comments on draft A, and a new section describing the proposed classroom innovation, with appropriate background. Participants are allowed to consult outside sources in developing their proposed classroom innovation. This draft is due during week 8 of the class.
- Draft C: A draft of the paper incorporating revisions to all previously submitted sections, including new progress on the mathematical problem. By this draft, this paper is expected to include background on the mathematical problem drawn from appropriate outside sources. This draft is due during week 11 of the class.

- Draft D: A draft of the paper incorporating revisions to all previously submitted sections, including new progress on the mathematical problem, along with a new section describing how the classroom innovation worked out (or is working out if it is still in progress). This draft is due during week 14 of the class.
- Final draft of the ARP. This draft is due during week 16 of the class.

At the end of the semester, in addition to turning in their written ARP, each participant has to make a presentation to the class, the instructor, and a second Master's committee member during finals week (week 17). The ARPs and presentations are evaluated by the instructor and second committee member to determine if they are sufficient to pass the ARP capstone requirement for the program. In some cases, participants are asked to make some final changes and/or corrections to their ARP before they can pass.

4 EXAMPLES OF PROJECTS AND TEACHING INNOVATIONS

One of the most challenging aspects of mentoring these projects is helping the teacher participants find appropriate mathematical topics that are at the right level for them to study, and that are connected to their own classes. As a teacher who uses inquiry-based methods in most of the classes I teach, I have experience with leading students through inquiry-based projects, and a collection of projects that I have used in various classes. In most of these other classes, students normally work through inquiry-based projects together in groups of three. (See [10], [11], and [12] for descriptions of some of these classes and how they incorporate inquiry-based group projects.) I have found, perhaps surprisingly, that a topic that I would use for a two to three week inquiry-based project in an undergraduate class is normally about the right difficulty level for a Master's level participant to investigate on their own for an ARP. Since they have to figure everything out themselves and the material isn't embedded in a larger course structure, everything takes much longer than it would as part of a regular class. However, there is also a lot more room for projects to go in unique and unexpected directions.

To illustrate what kinds of projects are possible, the research projects and classroom innovations of the 10 participants who completed the Mathematics ARP course with me during the first three semesters in which I taught it are summarized in Table 1. Many of these are discussed in greater detail below.

Most of the mathematical research topics that the participants looked at were based on inquiry-based projects drawn from a variety of different materials that I have used for courses I have taught. So, while the topics were new to the teachers investigating them, many of them had been previously used by students from other classes and had been proven to be both deep enough and accessible enough for participants to make good progress on. For example, participant 1's topic was based on an investigation from Chapter 4 of [14]; participants 2, 3, and 5 investigated questions based on problems from Chapters 6, 9, 11, and 18 of [6]; participants 4, 6, and 7 investigated questions from Chapter 1 of [11]; participant 10 investigated a problem from Chapter 7 of [11] which is a modified version of Problem 64 from [1]; participant 6 investigated questions from Chapter 13 of [10], which are also discussed in [9]; and participant 8 investigated problems from Chapter 6 and Appendix A.2 of [12]. Of course, given the greater flexibility of the ARP setting, many of the participants took these problems in unusual directions that are not normally explored in other classes.

In some cases, participants started the class with a preconceived idea of what general area or specific problem they wanted to study, or what they wanted to do for their classroom innovation, and then I worked with them to refine their ideas to make them work. In other cases, the participants had very little idea of what they wanted to do, and in these cases we usually started by talking about what they were teaching and tried to develop ideas from that. When a participant was having a lot of trouble coming up with ideas, I sometimes ended up giving then a specific list of possible topics to choose from, but when this happened, I always tried to create a list that was put together in response to the

	Mathematical research topic	Classroom innovation
1	Classification of all regular and semi-regular tessellations and polyhedra	Guided discovery of all regular tessellations and polyhedra using polydrons
2	Triangle congruence theorems on the sphere	Deciding if Side-Angle-Side is true on the sphere
3	Geometric solutions to quadratic equations	Construction of ellipses and derivation of the equation for an ellipse
4	Conics in taxicab geometry	Using Geogebra to explore constructions of ellipses
5	Proving that all planar isometries are reflections, rotations, translations or glide reflections which can be represented by matrix multiplication	Creating viral video calculator animations using matrix operations
6	π on non-Euclidean surfaces, especially cones	Measuring π on cones, spheres, and in taxicab geometry
7	Approximating π; exploring conics in taxicab geometry	Constructions of ellipses
8	Iterating functions, especially the logistic equation	Modeling population problems using iteration and spreadsheets
9	Developing spherical geometry from the ground up	Guided discovery of the formula for the area of a spherical triangle
10	Locus problems connecting geometry and algebra using GeoGebra	Exploring locus problems and conic sections using GeoGebra

Table 1. Participants' mathematical research topics and classroom innovations

participant's expressed interests.

Most of the topics that participants have explored have come from the areas of geometry, algebra, and pre-calculus, with a particular emphasis on geometry. This is because these are the topics that most high school teachers teach, and, of these, geometry seems to be a particularly rich source of problems that are deep enough for participants to explore, while still being connected to the high school curriculum. Geometry is also the topic that I have taught the most, and in which I therefore have the largest store of appropriate topics to propose.

Both the research projects and the teaching innovations that the various participants have done have varied quite a bit in their depth. This is in part because I have tried to steer each participant to a project that is at an appropriate level for that participant, and also because some participants get further with their projects than others. Likewise, the teaching innovations have also varied in scope since they are constrained both by what the individual participants feel comfortable doing and also by how flexible their classes and schools are in being able to accommodate a teaching innovation. Some teaching innovations have only taken a day or two of class time, while others have taken up to a month. However, some of the projects done on relatively easier topics by less mathematically mature participants were among the most successful and transformative, because they marked the participant's first real experience of discovering mathematics successfully on their own. Similarly, some of the smaller teaching innovations may have impacted the participants' teaching as much as the larger-scale teaching innovations, because they were changes that the participants were comfortable with and likely to try again in the future.

4.1 PROJECT SUMMARIES

This subsection gives more detailed descriptions of some of the participants' projects in order to illustrate how their mathematical inquiries informed the inquiry-based classroom innovations that they did with their own students. It is interesting to note that about half of the participants chose to ask their own students to explore questions that came directly from some part of their own inquiries.

One participant who developed a classroom innovation that was directly taken from his mathematical research was participant 1, whose mathematical research was to classify all of the regular and semi-regular tessellations and polyhedra and to prove that he had found all the possibilities. He first explored these using plastic snap-together manipulative polygons called polydrons, and then carefully identified all possible tessellations and polyhedra using mathematical arguments in conjunction with an exhaustive computer search done using Excel spreadsheets. For his teaching project, he created two guided discovery lessons for his high school geometry classes in which his students, working in groups, duplicated a small part of his project by using worksheets he developed that led them to discover and classify all of the regular tessellations and polyhedra using polydrons. Reflecting on the innovation in his ARP, participant 1 made clear that teaching this lesson pushed him out of his normal comfort zone, but that he saw clear benefits. He wrote, "For my classroom innovation I created two guided-inquiry lessons . . . I felt this lesson format would be an innovative teaching method for me because it required me to allow the students more control of the lesson and to discover rules for themselves. The format of these lessons forced me to fight my own natural tendency to step in and pull the students towards the answer. My role in these lessons was less of an instructor and more of a guide and facilitator. . . . I was very pleased with the student attitude and involvement in the lesson. The students were excited to use manipulatives and work in groups. The discovery style of the lesson kept the students interested and involved from the start of the lessons to the finish."

A second person who chose to create a classroom innovation taken directly from part of his own mathematical explorations was participant 8. For his mathematical research, he investigated iteration and discrete dynamical systems using spreadsheets, focusing in particular on iteration of the logistic function and using discrete dynamical systems to understand modeling problems. For his teaching innovation, he did a three day enrichment activity with some of his Algebra II students, asking them to explore three modeling problems using spreadsheets. The first problem asked students to model exponential growth of an animal population; the second asked them to model exponential growth with constant harvesting; and the third asked them to explore iterating the logistic equation. All three problems came directly from his own explorations. Participant 8 felt that while his normal teaching practice was built around the common core standards, it was focused primarily on the content standards, and that he was not giving enough attention to addressing the standards for mathematical practice. He therefore intentionally designed his innovation around these standards. Reflecting in his ARP, he wrote, "Overall, this innovation has helped me to understand where I can continue to grow as a teacher. I need to focus on finding ways to engage all of my students in interesting, real-world problems that highlight key mathematical practices.... I want to create a classroom based on what the National Council of Teachers of Mathematics discusses in their recent book on teaching mathematics, *Principles to Actions*. In it, they describe the mathematics teaching practices, such as, 'implement tasks that promote reasoning and problem solving,' and, 'facilitate meaningful mathematical discourse.' By implementing the mathematics teaching practices with fidelity and consistency in my classroom, I can help my students grow into great mathematical thinkers who will be able to accomplish great things in their future."

Participant 6 went even farther in incorporating the topic of her own mathematical exploration into her classroom innovation. She not only had her students explore a question that she was investigating, but even allowed two groups to explore it on two surfaces that she herself hadn't explored, so that she didn't know ahead of time what they might find. Her mathematical research was to investigate formulas for the value(s) of π in taxicab geometry and on cones. For her classroom innovation, she spent two weeks during her applied mathematics course in which she divided her eight person class into three groups. Each group picked a non-Euclidean surface and investigated π on that surface. One group looked at cones, one looked at spheres, and the last group looked at cylinders, even though she herself had only explored π on cones. At the end of the unit, each group wrote up their findings and also made a formal presentation of them to the whole class. This was one of the riskiest innovations, because it put the participant far outside her normal comfort zone in the way she taught the unit, and she even gave up the control of knowing what her students might find. However, it turned out to also be one of the most rewarding and transformative innovations. Participant 6 reflected in her ARP that "[a]s students were working on this project, it was very hard for me to just step back and let them have full control of what they were working on without telling them what to do next.... Despite my difficulty with letting students pace themselves through their projects, I really did see the benefits of having students working together and truly understanding what they were doing. ... The questions my students were debating about amongst their groups really allowed me to see the great things that happen when group work occurs correctly in the classroom. When students were asked to make their own predictions, test them and analyze the results they learned so much more than they would have if I just explained to them what happened. I know the push from administrators and the state is to include more collaboration in our classrooms but before this project I really struggled with myself whether it was benefitting the students or just another check off of my observation checklist.... During this process, the students definitely learned what it was like to be a mathematician and what it was like to investigate how or why something happens. I do not think they would have had the same conversations or understandings if I had given them step-by-step directions of what to do during their investigation process. It was really good for me as a teacher to see students accomplish this and to have those 'I get it' moments.... I am no longer intimidated about giving students free rein in the classroom but instead have learned ways to structure this free rein so students will be more successful."

The person, however, who went the farthest in radically transforming his classroom was participant 5. He is a teacher who was already using inquiry in his classroom, and worked at a school that was using the inquiry-friendly Interactive Mathematics Program (IMP) curricula [5]. For his classroom innovation, he modified a unit from his fifth-year IMP class to base it around an open-ended project, and restructured his classroom practice during the several weeks they were working on this project so that it was almost entirely centered around students' explorations and questions rather than being primarily teacher-directed. The student project was to create animated "viral" videos on their calculators, using matrix operations to achieve the animations. This participant wrote that his new approach increased student "ownership and buy-in," led to greater student interest and better student questions, and that class time better met students' needs. In fact, it was so successful that the following year, he radically redesigned his entire class to be run in the open-ended project-based style of this innovation, as part of a new project-centered learning community at his school called the "Mosaic Collective." In his ARP, participant 5 wrote,"I want to focus on the relevance just as much as the rigor of the mathematics in order to have more motivated learners that will desire to know more rather than being told to learn more. I want to move students toward habits of questioning rather than habits of compliance. It is to this end that I am implementing a project-based approach. In order to successfully create this culture a shift is necessary in the role of the teacher. The teacher cannot be viewed as the almighty holder of math knowledge; instead, we should be the crafter and guide on a journey through experiences with mathematics and the classroom becomes a social network of learning and not a funnel for pouring content into students."

On the other hand, several participants did much smaller teaching innovations that were confined to a few days of class and involved much smaller changes. Some participants did smaller innovations because that was what they felt comfortable with, while other participants did smaller innovations because their teaching situations were more constrained and they didn't have the flexibility make large changes to their teaching. However, although they were smaller, these innovations still tended to be more inquiry-based than the participants' usual practice. In addition, because they were small, they were changes that could be easily used again in the future, and participants expressed their intention to do so. For example, participants 4 and 7 both explored the possible shapes of circles, ellipses, and hyperbolas in taxicab geometry using GeoGebra (during two different semesters), and both did relatively small teaching innovations. Participant 4 developed computer labs to have her students create Euclidean ellipses and hyperbolas using Geogebra in her Algebra II classes. This was the first time she had used GeoGebra or any kind of dynamic geometry software, and she became an enthusiastic advocate for using this technology in the classroom. In her ARP, she wrote, "The math research and classroom innovation I have done this semester resulted in me trying new things and stepping out of my comfort zone. It pushed me to learn, use, and teach with the geometry technology software, Geogebra. In the future I will be trying to plan lessons that include this software when and where I can because the students enjoyed it." Likewise, participant 7 created a one-day worksheet-based guided inquiry lesson in her college algebra class focused on creating ellipses physically using string and connecting this to the equation for an ellipse. In her ARP, she wrote, "Some students really liked this activity and asked me if they could do similar types of activity for the next lesson. So going forward, I hope to design more student-centered lessons that would stimulate discussions and thinking amongst students. Also, more interaction with students helped me understand their thought process. So knowing where they stand will certainly help me to redesign intuitive student-centered lessons."

Of course, some participants created classroom innovations that lay between these two extremes. One such person was participant 10, who decided to investigate modeling problems involving loci with GeoGebra in order to relate algebra and geometry. He ended up looking at a paper folding problem (Problem 64 from [1]), a problem about the shape of road on which a bicycle with square tires could ride smoothly, and some problems about Reuleaux triangles. For his classroom innovation with Algebra II students, he put together guided discovery lessons on loci connected to conic sections using GeoGebra. This classroom innovation was on a similar topic to those of participants 4 and 7, but was quite a bit more extensive, with more ambitious goals. Participant 10 wanted to use his classroom innovation to tie algebra and geometry together while at the same time focusing on the common core standards for mathematical practice, just as participant 8 did. In his ARP, he reflected that "[l]ocus problems bridge the gap between geometry and algebra. At the secondary level locus problems connect the content learned from year to year and are perfect for challenging students with rigorous problems that engage all of the mathematical practices outlined in the Common Core State Standards. . . . My experiences in the mathematical explorations, research, and classroom innovation above have opened my eyes to new problem solving schema, new technology skills, and new possibilities for classroom activities. It was both a struggle and a pleasure that required more of my mental faculties than anything in recent memory. I feel like I have found a new hobby and I intend to continue exploring locus problems in the future."

Taking all of these participants' reflections together, we see quite a bit of evidence of what is sought in action research: "transformative change through the simultaneous process of taking action and doing research, which are linked together by critical reflection."[15] In this case, there is an additional link between the participants' mathematical research and their classroom innovations, which are another form of research. Their mathematical inquiries give them the means and desire to try inquiry-based activities with their own students, which, when successful, will hopefully inspire them to continue to try further innovations and inquiry-based teaching in the future.

5 CONCLUSION

In the end, this format for a capstone requirement for a Master's program for in-service secondary mathematics teachers has worked extremely well. It gives the teachers an extended, meaningful experience conducting mathematical research that is connected to their own classrooms. In most cases, the participants have used this research as a starting point for designing lessons for their own classrooms that are also inquiry-based. Thus, it seems that a great way to get teachers to incorporate inquiry-based lessons into their teaching is to have them engage in inquiry themselves and then ask them to design lessons for their own classrooms related to the areas they have been researching.

This was summed up well by participant 3 in his ARP, in which he wrote:

> Through my experiences, both in researching mathematical ideas myself and in designing innovations for class, I have learned a lot. The innovations came from my desire to have students struggle through problems on their own, which was an experience I got to have alongside them as I worked on my own research. In addition, this project gave me the opportunity to implement innovations in class that I would not have had the courage or motivation to do otherwise. I hope that this will give me confidence and competence moving forward to implement other innovations into the classes I teach.

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BIOGRAPHICAL SKETCH

Nathaniel Miller attended Princeton University, then went on to complete a Ph.D. at Cornell University, where he worked with David Henderson, a great advocate for inquiry-based teaching and learning. He is now

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