Classification of Activity Patterns in Small Neural Networks in terms of Network Architecture

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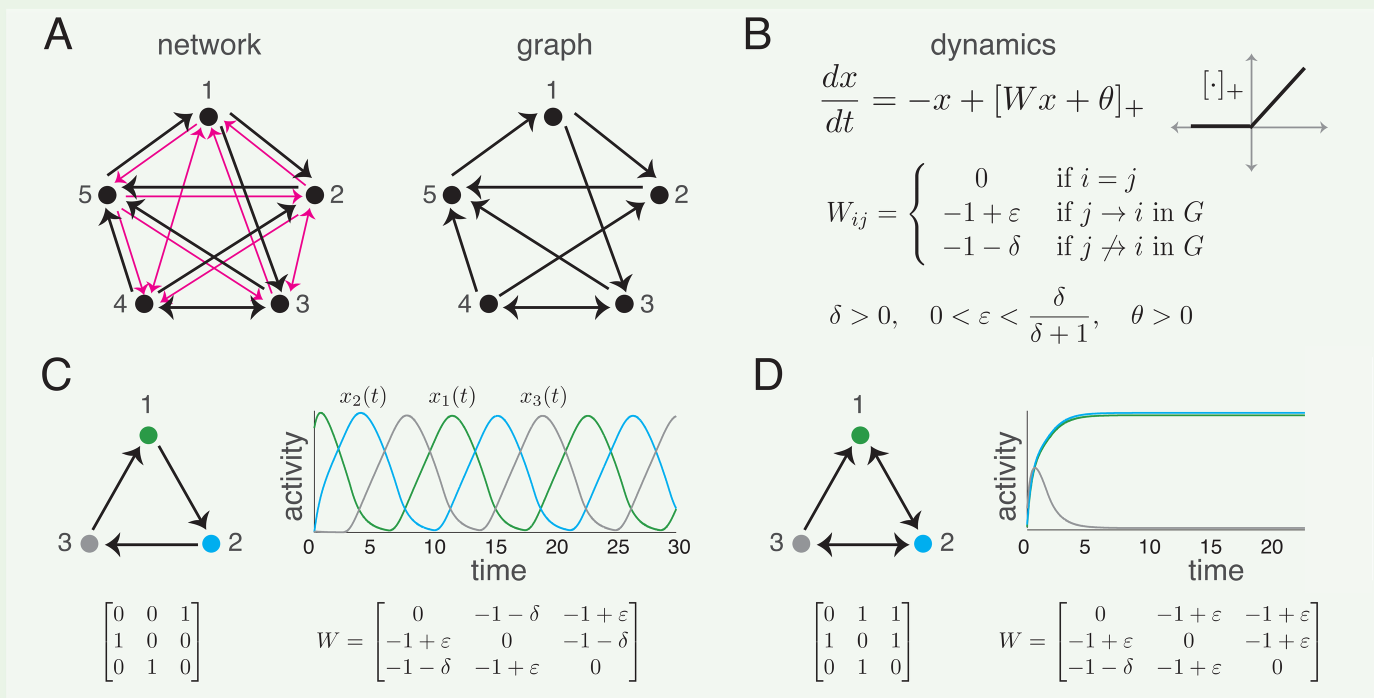
⟷ Combinatorial threshold-linear networks (CTLN) ⟷

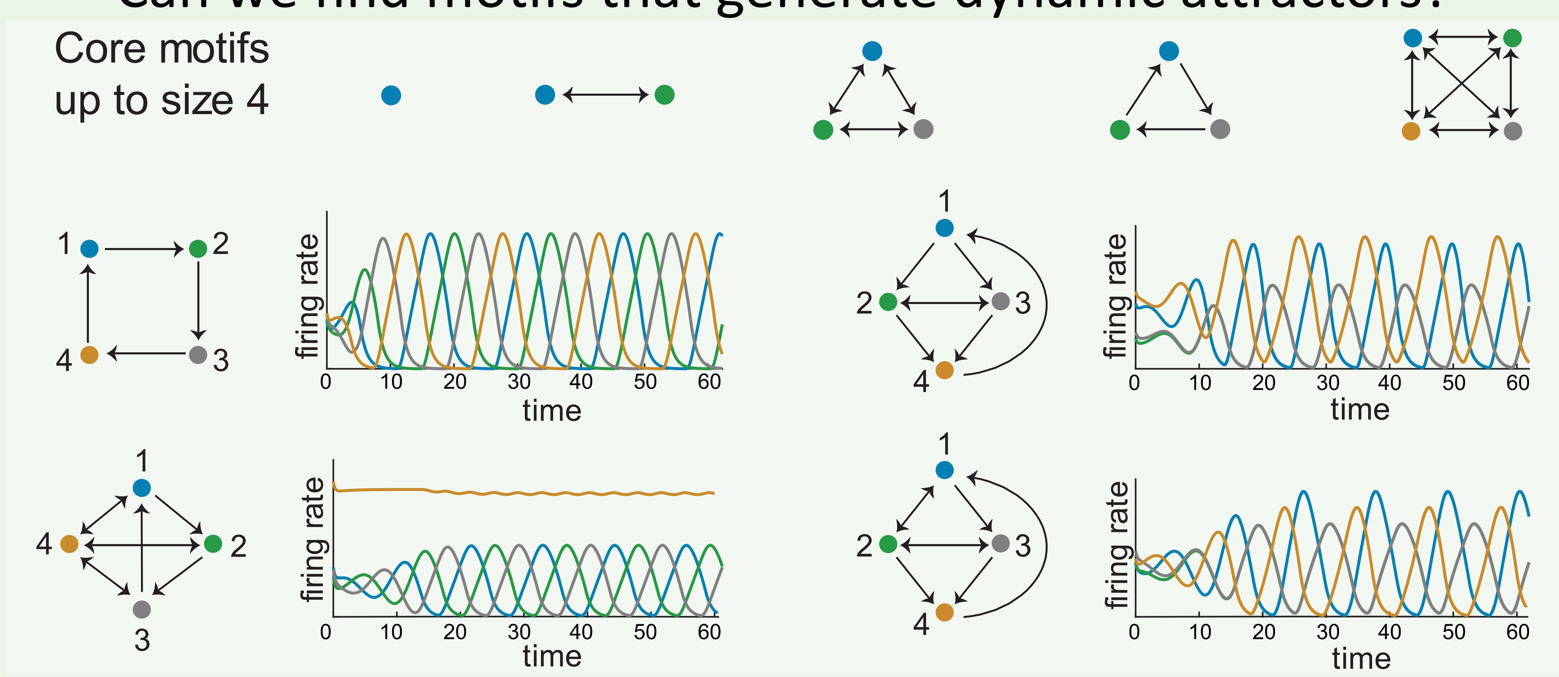


Unless otherwise noted, ϵ=.51, δ=1.76, ϑ=1 in all simulations. Thus, differences in dynamics are due only to differences in the graph G.

⟶ Motivating Questions: ⟵

How does network connectivity shape emergent dynamics?

Can we find motifs that generate dynamic attractors?



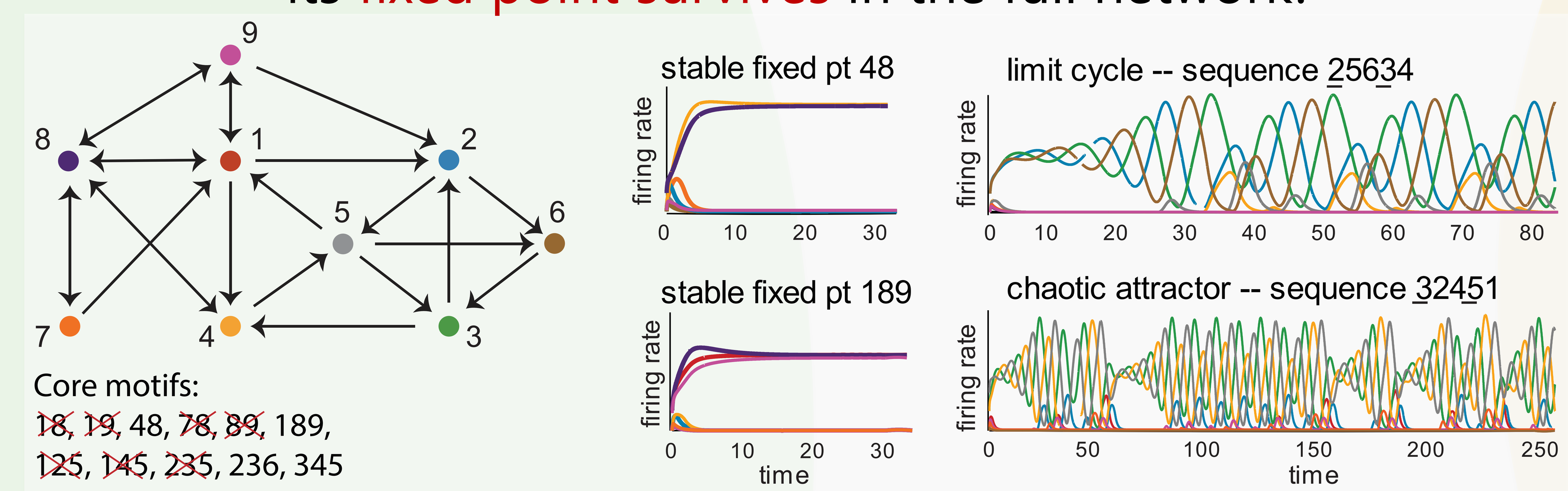
A core motif is a subgraph $G|_\sigma$ with a unique fixed point $\mathbf{x}^*$ that has full support, i.e. $x_i^* > 0$ for all $i \in \sigma$.

Graphs	Survives addition of *k*	Does not survive addition of *k*
4-cycle	at most one edge to k	at least two edges to k
4-ufd	at most two edges to k	at least three edges to k
4-clique	at most three edges to k	all four edges to k
fusion 3-cycle	if 4 → k, then at most one edge from the 3-cycle 123 to k; if 4 ↛ k, then all edges from 123 to k allowed	4 → k, and at least two edges from the 3-cycle 123 to k
4-cycu	at most one edge to k; or any pair of edges from {1,2,3} to k; or 2,4 → k; or 3,4 → k	at least three edges to k; or 1,4 → k

A set of graph rules determines whether the fixed point of the core motif survives the addition of a node *k*. The fixed point survives in the full network if it survives the addition of each external node individually.

⟵ Rule of thumb: ⟶

Every core motif has an associated attractor
if and only if
its fixed point survives in the full network.

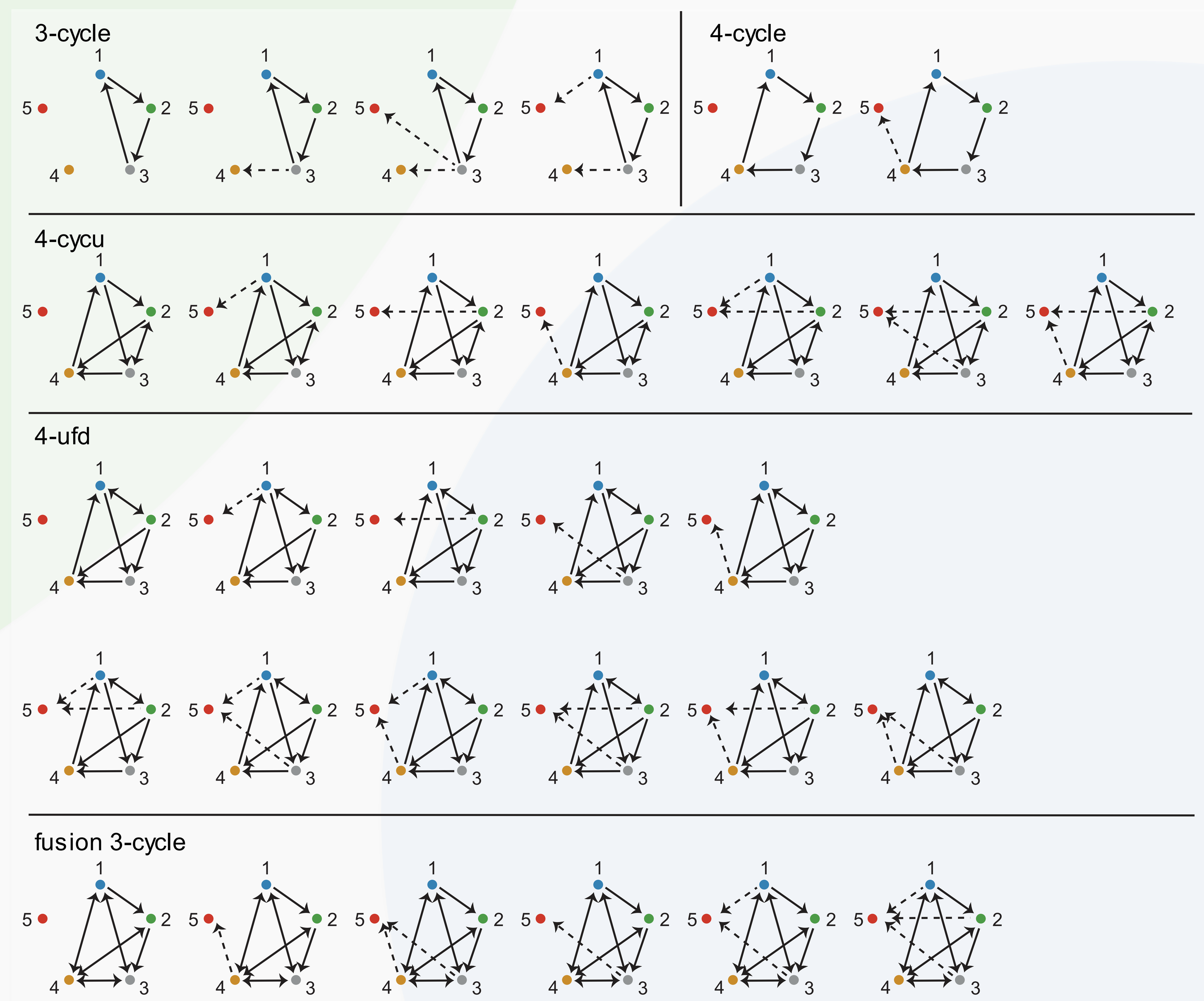


This n=9 graph contains 11 core motifs, but only 4 have a fixed point that survives in the entire network. Thus, 4 attractors are produced.

⟶ Testing the rule of thumb ⟵

Of the total 9608 graphs of size 5, the Rule of Thumb holds for all but 19 graphs.

We focus on classifying the dynamic attractors of the 1014 graphs that contain a core motif of size up to 4 that produces a dynamic attractor.

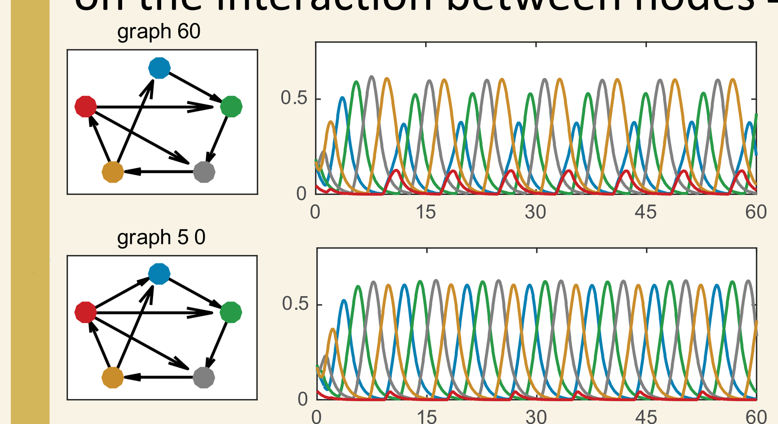


There are 5 core motifs up to size 4 that produce dynamic attractors. The table above shows all the core-periphery classes for graphs of size 5. Each class consists of a core motif and a set of outgoing edges to node 5 such that the fixed point of the core motif survives.

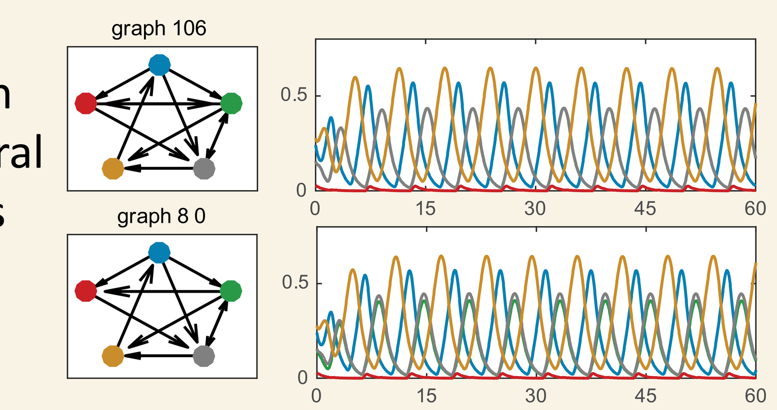
⟷ Summary of n=5 classification of attractors ⟷

The core-periphery classes largely predict the structure of the dynamic attractors of the graphs of size 5, irrespective of the back edges from node 5 to the core motif.

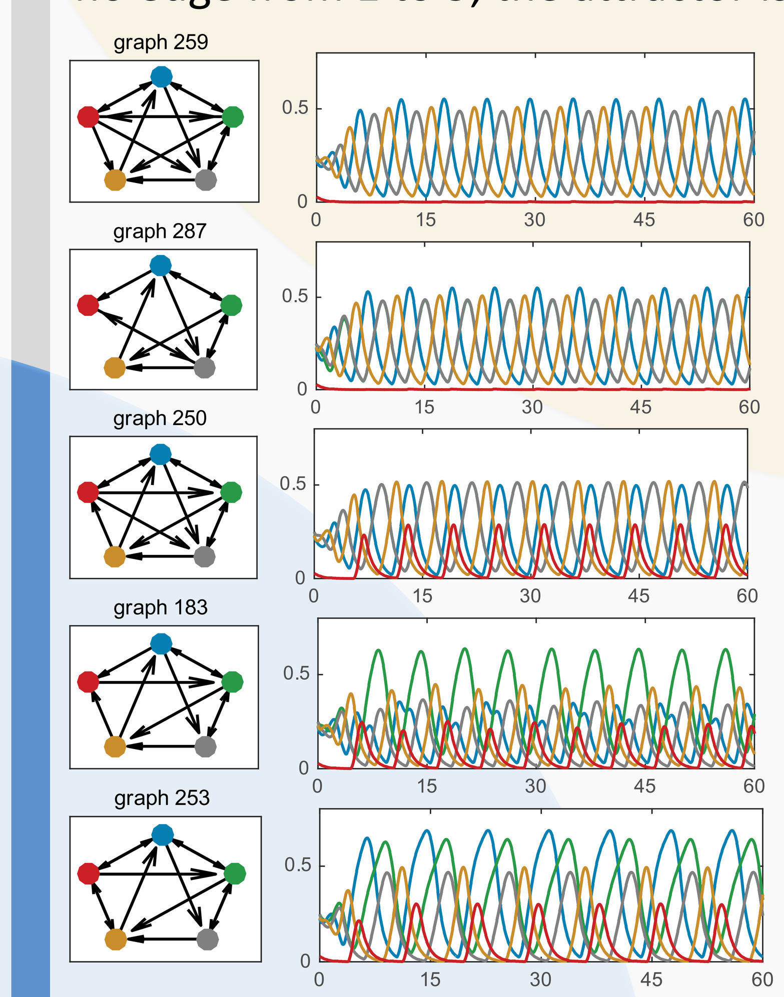
3-cycle: The core-periphery classes perfectly predict the structure of core attractors for embedded 3-cycles, except there is a split in the third class based on the interaction between nodes 4 and 5.



4-cycle: There is slight variation between the attractors in this class based on the height of the peripheral node depending on whether 5 sends an edge to 1.



4-cycu: There are some discrepancies between the attractors of the graphs when the peripheral node sends an edge to node 2 or 3 that breaks the symmetry. Additionally, whenever there is no edge from 1 to 5, the attractor is missing.



4-ufd: This class has significantly more variation between attractors depending on back edges from the peripheral node.

- There is no firing of node 5 when it receives 1 or no edges
- Low firing when node 5 receives from
- 1 & 2, 1 & 3, or 2 & 3
- Higher firing when 5 receives from 1 & 4
- A back edge from 5 to 2 or 3 can break the symmetry
- When there is no edge from 5 to 2, then the attractor is missing

Fusion 3-cycle: This class is perfectly predicted by the core-periphery classes.

⟶ Key Takeaways ⟵

1. Surviving core motifs give attractors.
2. Core-periphery classes predict general features of attractors, and back edges from peripheral nodes often have little effect.
3. Back edges from the periphery to the core *do* matter when they break symmetry between nodes in the core.

C. Parmelee, J. Paik, D. Olds, K. Morrison, C. Curto. *Classification of dynamic attractors in small inhibition-dominated threshold-linear networks*. In prep.

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