

Linear-Fractional Dynamics on the Riemann Sphere with Geogebra

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➤ Introduction

This PowerPoint shows my process of my directed study. My research topic is carried out on the complex plane, mainly including what kind of property between two mirrors with different relations have on a complex plane. The goal is to find out the property of the orbit.

➤ Two Line Mirrors

Cases

When two lines are not perpendicular.

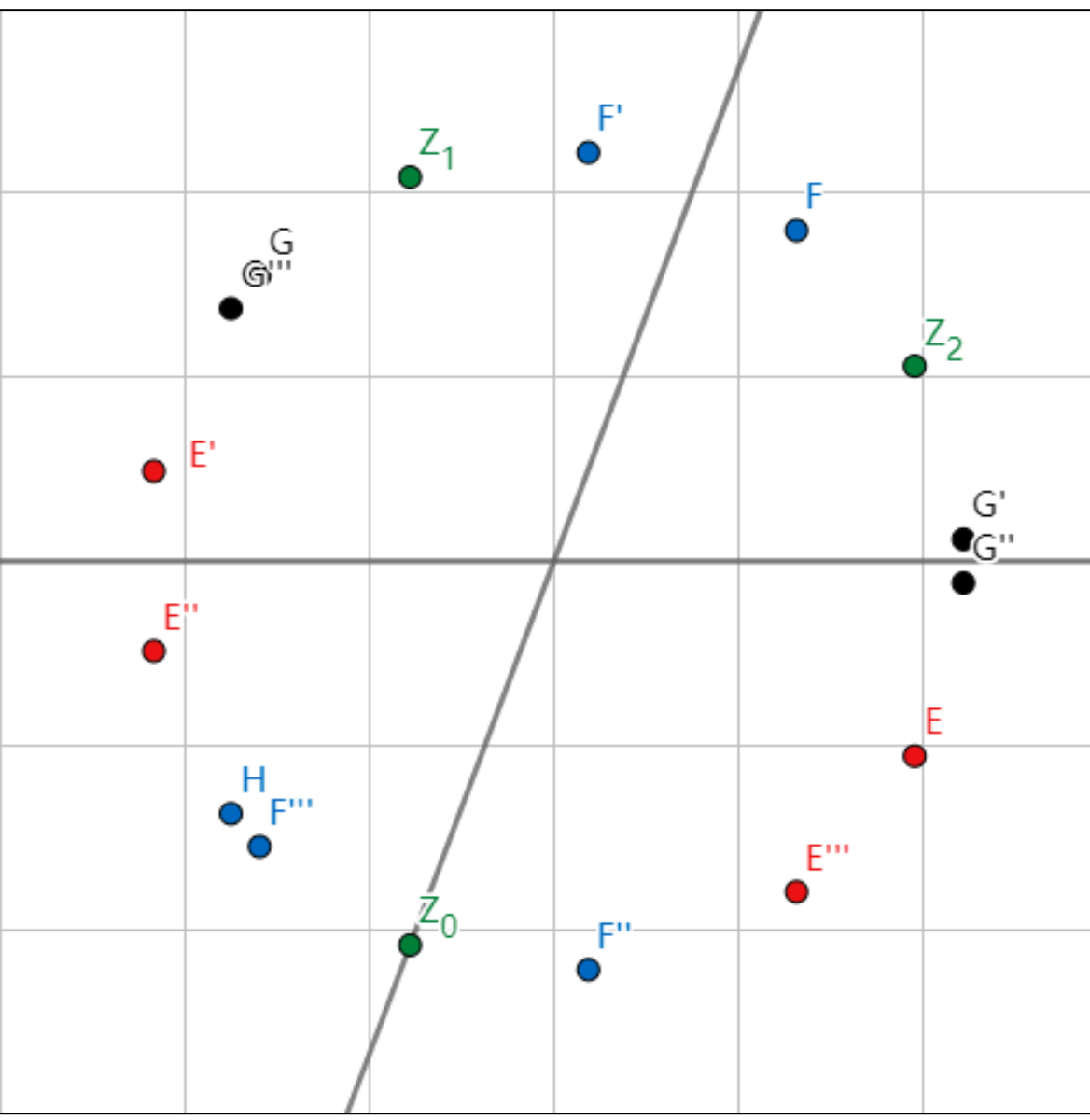
The orbit is a circle, center is the intersect point of L1 and L2.

When two lines are perpendicular.

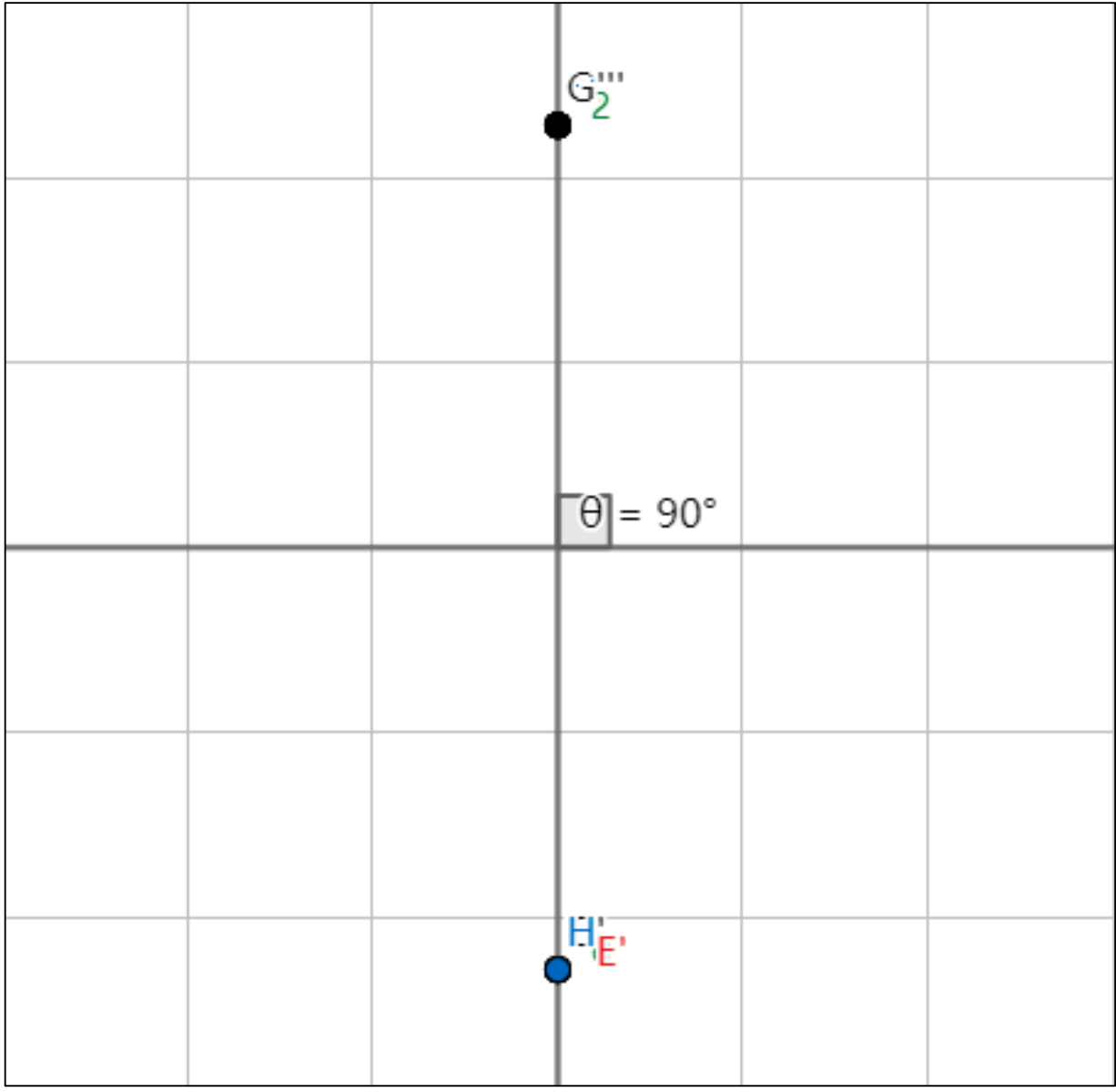
When two lines are perpendicular, which means $\theta=90^\circ$. Under this condition, the orbit is two points.

Property

The angle between two points is twice than the angle between two line mirrors, which means $\alpha=2\theta$.



When two lines are not perpendicular.



When two lines are perpendicular.

➤ A Line and A Circle Mirror

The line and the circle are not touching.

After multiples times reflection, the orbit is a circle.

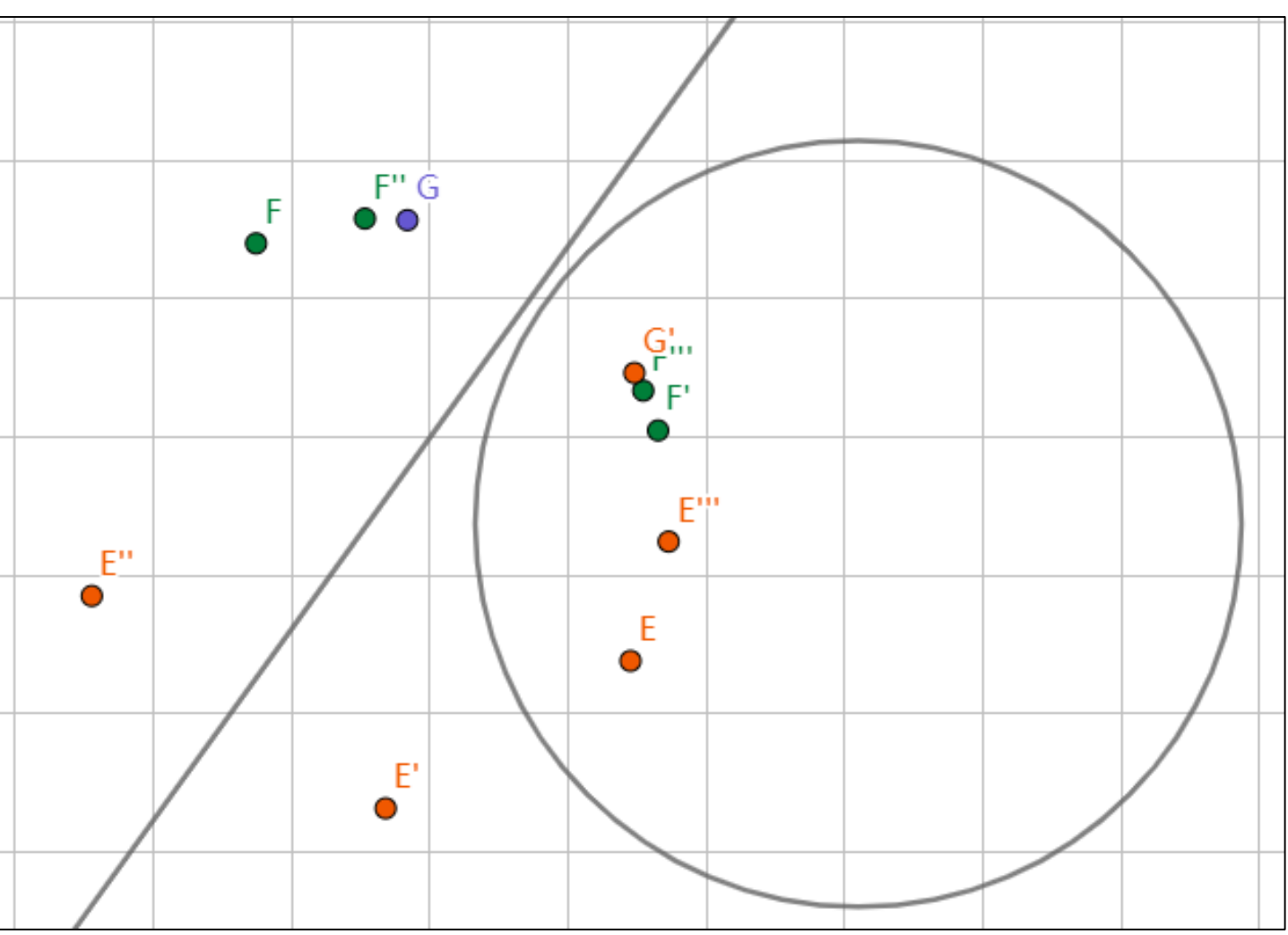
The line and the circle are intersect or tangent.

Under this condition, the orbit is a circle.

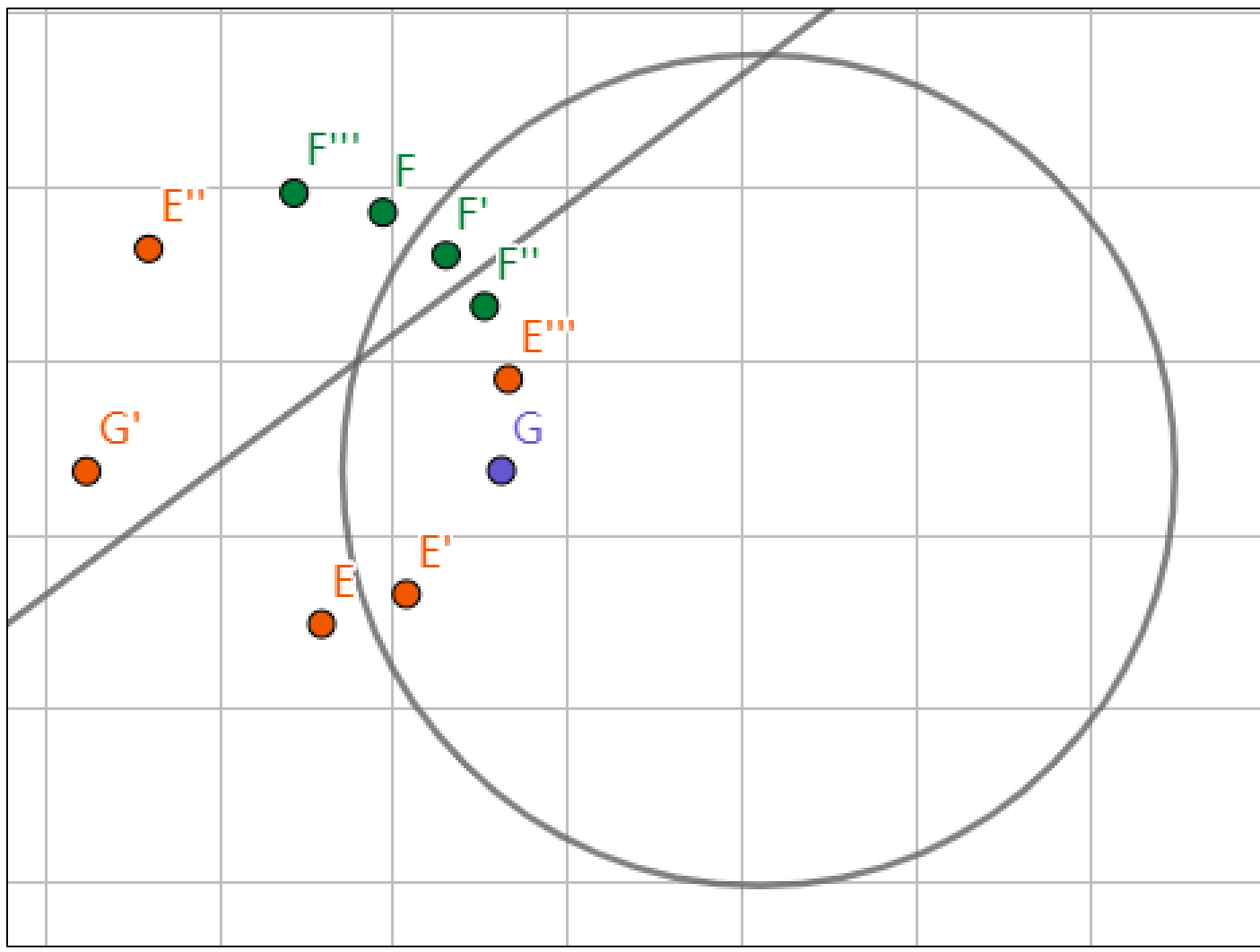
Property

No matter under what condition, the orbit is a circle.

(The position of the point does not effect the result.)



Two mirrors are not touching



Two mirrors are intersect or tangent

➤ Geobebra Model & General Algebra Idea

Draw two mirror L1 and L2. Named the reflection over L1 T1, and the reflection over L2 T2. Consider T1oT2 as one reflection.

$$T1(z) = \frac{a1\bar{z}+b1}{c1\bar{z}+d1} \quad T2(z) = \frac{a2\bar{z} + b2}{c2\bar{z} + d2}$$
$$T3(z) = T1(z)oT2(z) = \frac{(a1a2 + b1c1)z + (a1b2 + b1c2)}{(a2c1 + c2d2)z + (c1b2 + d1d2)} = \frac{Az + B}{Cz + D}$$

➤ Two Circle Mirrors

Two circle mirrors are not touching.

Under this condition, the orbit is always an arc between two circle mirrors.

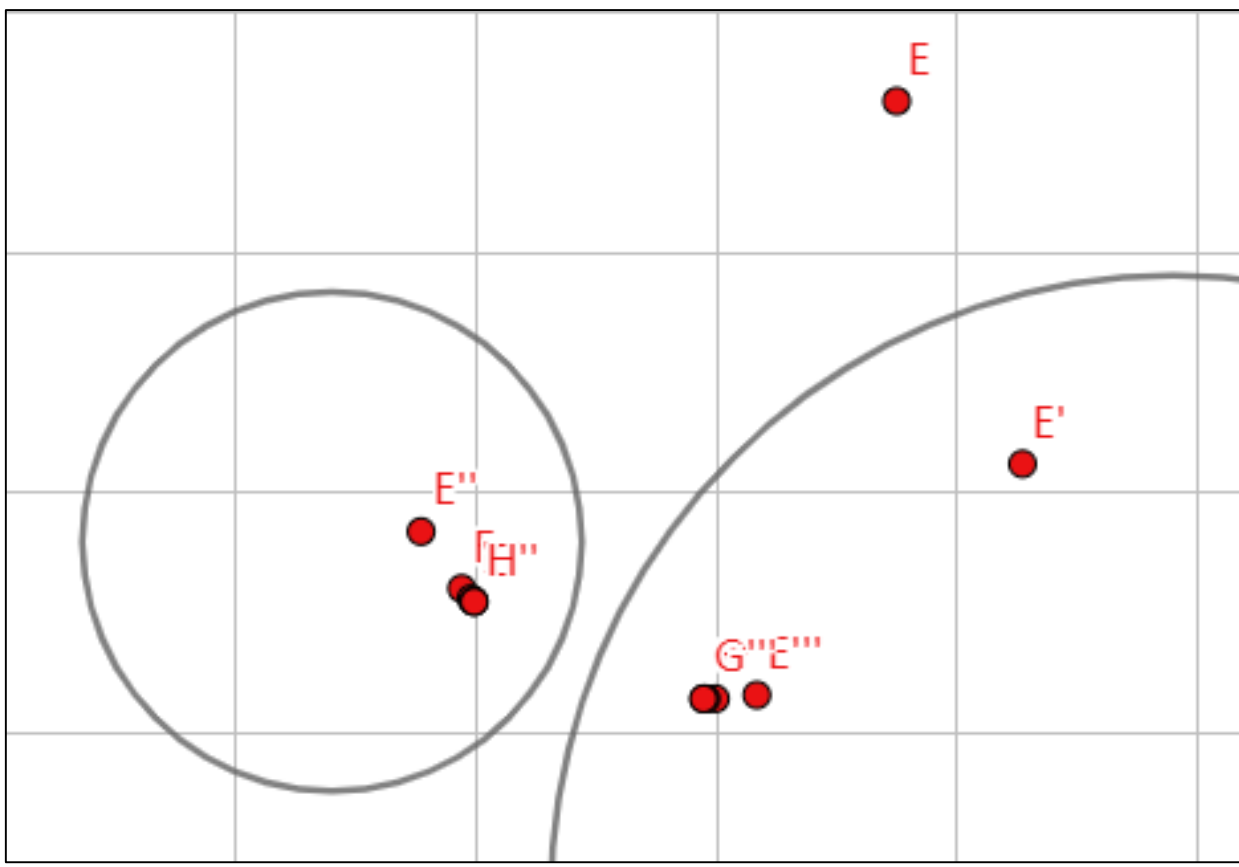
Two circle mirrors are tangent.

The orbit will always create arc between two circle mirrors.

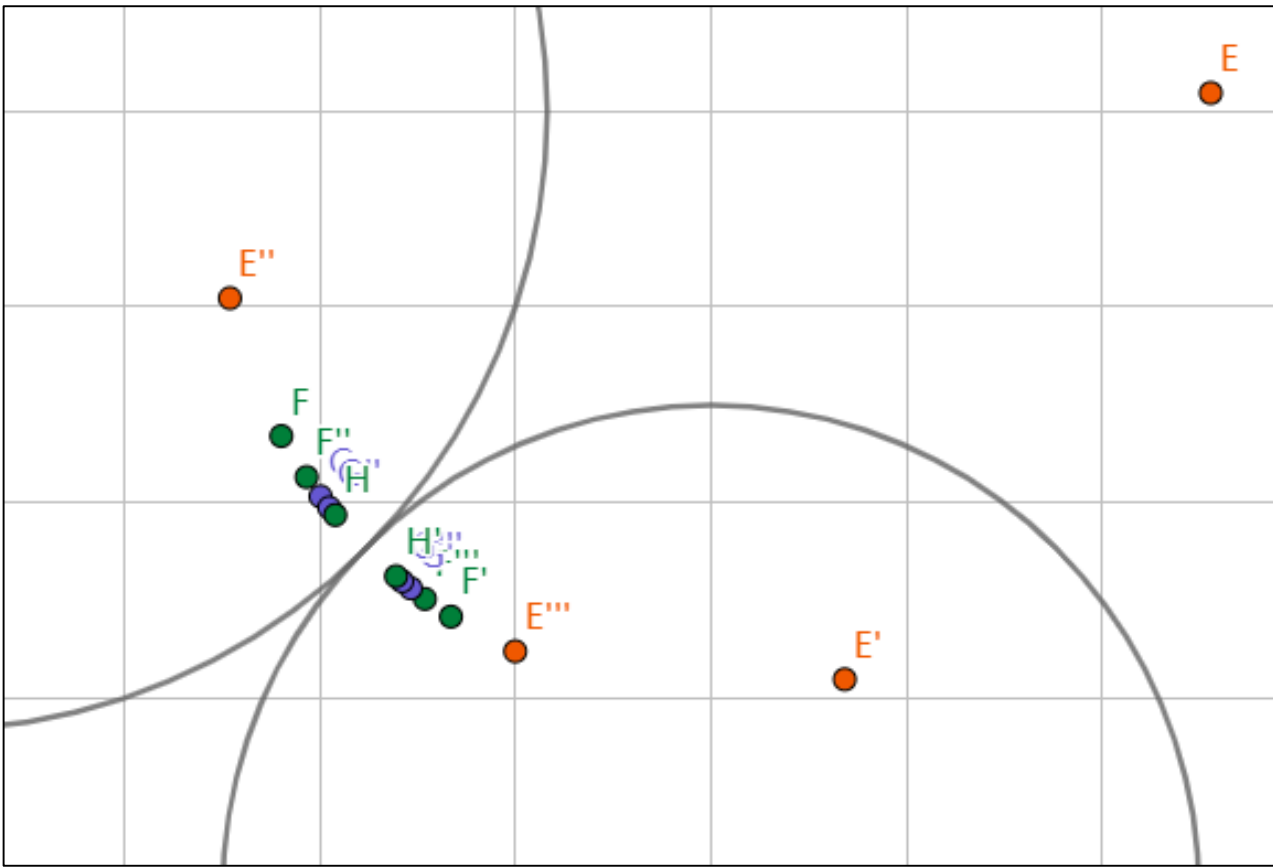
Two circle mirrors are intersect.

Under this condition, the orbit is a circle.

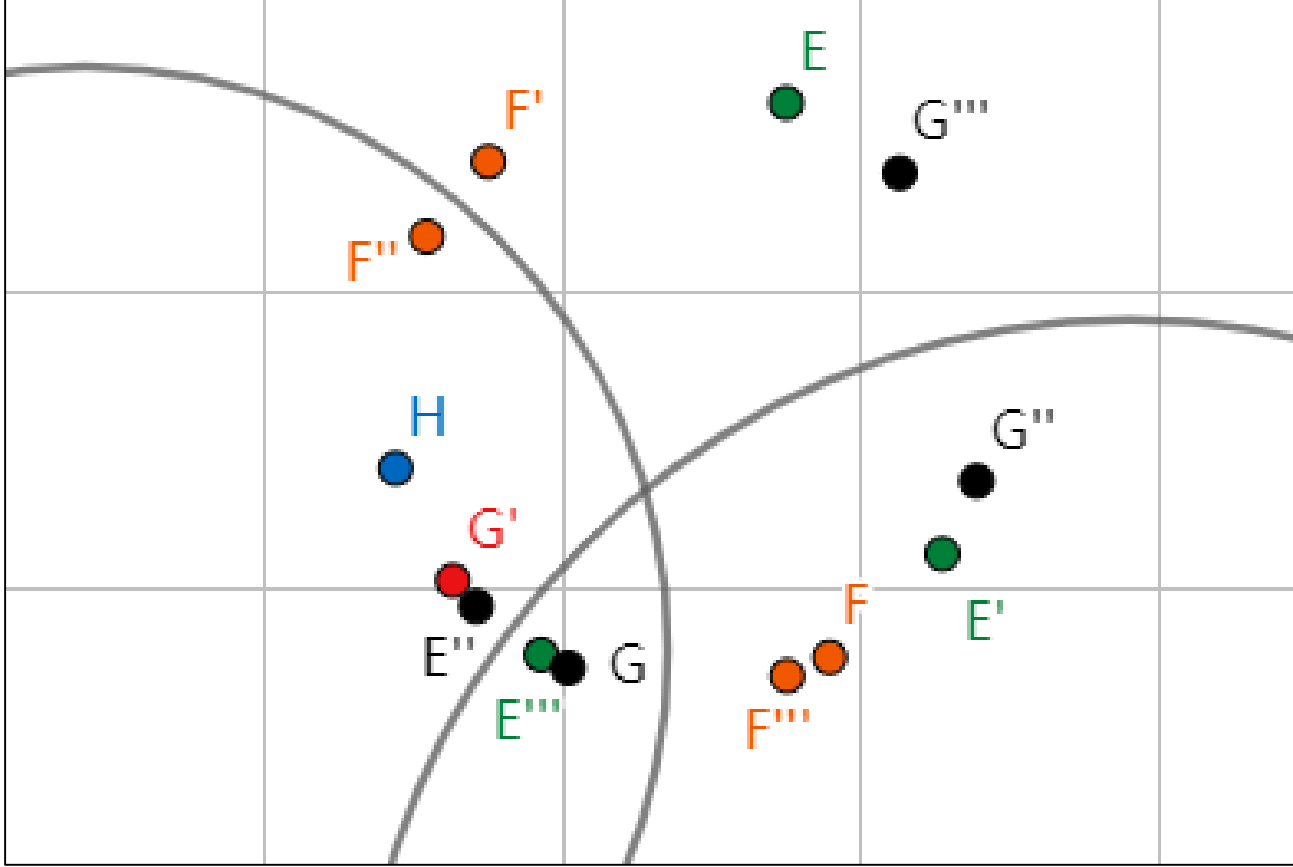
(the position of the point does not effect the result.)



Two circles are not touching.



Two circles are tangent.



Two circles are intersect.

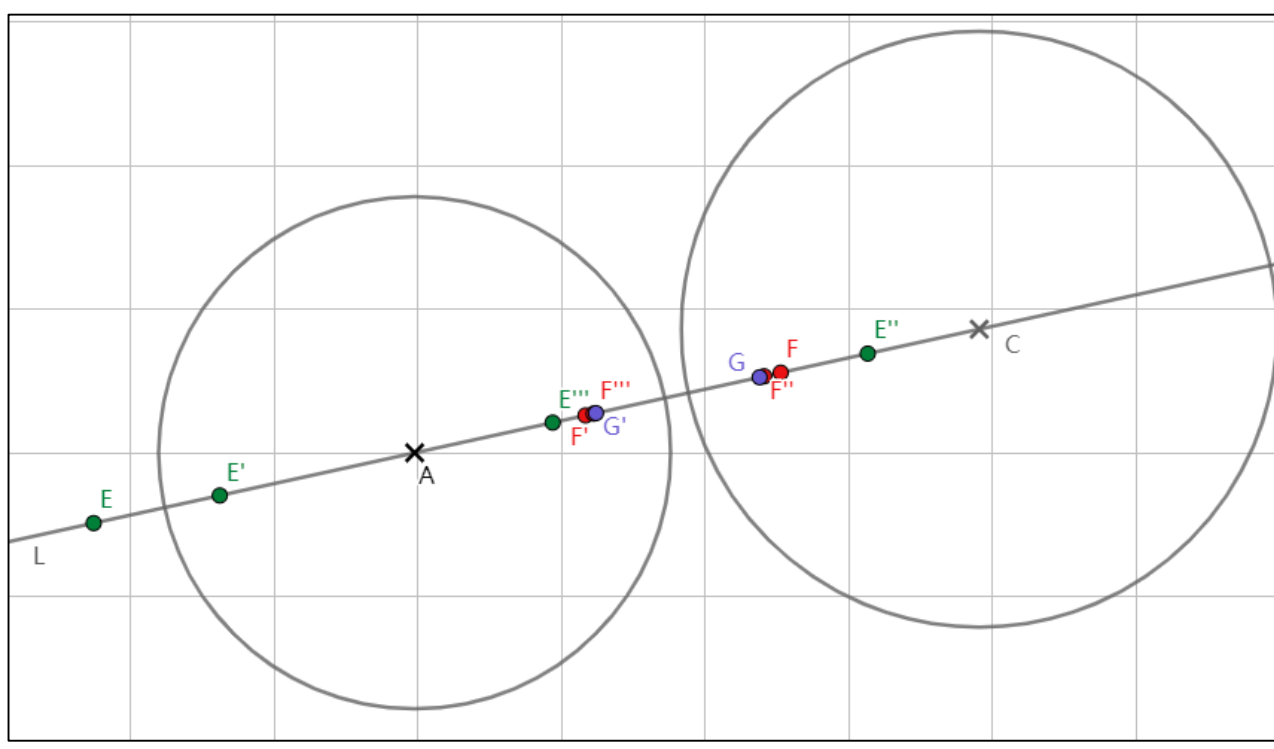
➤ The line L pass through the centers of two circle mirrors

Let T3 denote the double reflection map in two circles.

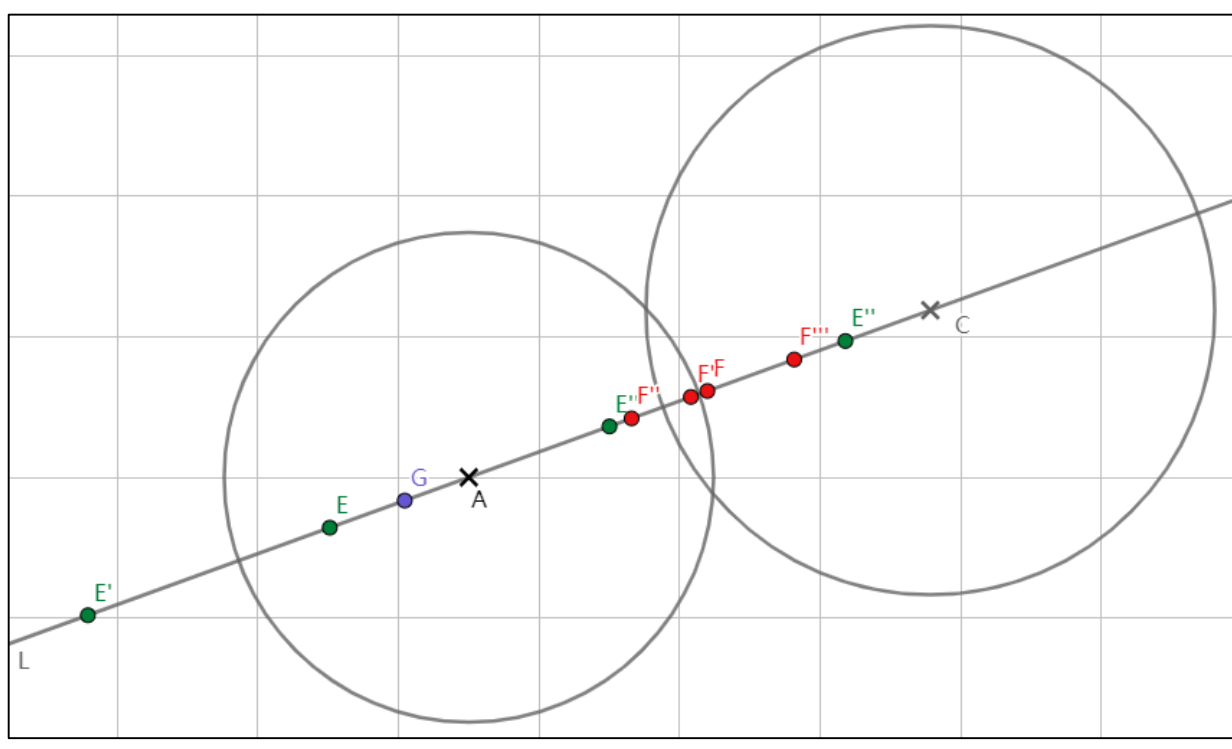
Let line L be the line that pass through their centers.

Then every point on L is mapped into L by each circular reflection.

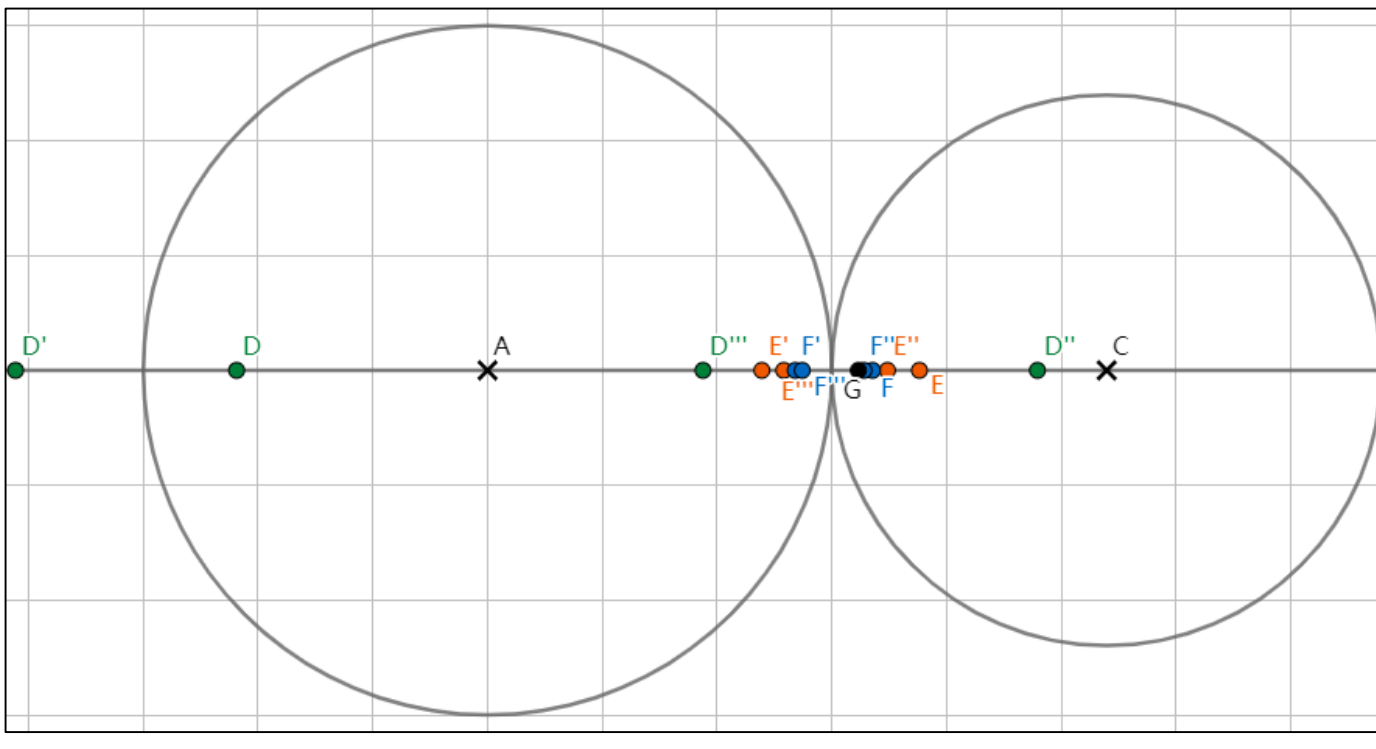
Thus the double reflection map T3 also maps L into itself.



Two circles are not touching.



Two circles are tangent.



Two circles are intersect.

➤ Conclusion

Types of mirror	Position relationship	Orbits
Two line mirrors	Two lines are not perpendicular	Circle
	Two lines are perpendicular	Two points
A line mirror and a circle mirror	Not touching	Circle
	Intersect or Tangent	Circle
Two circle mirrors	Not touching	Arc
	Tangent	Arc
	Intersect	Circle
The line L pass through the centers of two circle mirrors	N/A	Maps L into itself