Name (print): ___________________________________

Instructions: There are 8 numbered questions worth a total of 250 points. Make sure you have a complete copy of the exam before beginning. *You must show all of your work to receive full credit for every question.* Respond with clear, complete sentences when asked to explain your reasoning or justify your answer.

Additional space is provided on a blank page at the end of the exam. If you want any work on that page graded, write the question number next to it and write “See extra work.” on the space provided in the exam.

Check your class section:

_____ Heiny, MWF 10:10am, T 11:00am

_____ Morrison, MTWF 8:00am

_____ Morrison, MWF 10:10am, T 11:00am

_____ Oehrtman, MWF 1:25pm, T 11:00am

_____ Raish, MTWF 8:00am
1. The population of Nicaragua was 3.6 million at the beginning of 1990 and growing at an annual rate of 3.4% per year. Let $P$ be the population of Nicaragua in millions of people and $t$ be time in years since 1990.

   a. Express $P$ as a function of $t$.

   b. What is the doubling time of the population?

   c. Find the continuous rate at which the population of Nicaragua is growing. Recall that the continuous rate is the value of $k$ such that $P(t) = P_0 e^{kt}$ and $\frac{dP}{dt} = kP$. 
2. Consider the function $f$ given by the following graph of $y = f(x)$:

Find the following values (write “dne” if they do not exist and $-\infty$ and $+\infty$ when appropriate):

\[
\begin{align*}
\lim_{x \to -3^-} f(x) &= \hspace{2cm} \lim_{x \to -3^+} f(x) &= \hspace{2cm} \lim_{x \to -3} f(x) &= \hspace{2cm} f(-3) = \\
\lim_{x \to -1^-} f(x) &= \hspace{2cm} \lim_{x \to -1^+} f(x) &= \hspace{2cm} \lim_{x \to -1} f(x) &= \hspace{2cm} f(-1) = \\
\lim_{x \to 0^-} f(x) &= \hspace{2cm} \lim_{x \to 0^+} f(x) &= \hspace{2cm} \lim_{x \to 0} f(x) &= \hspace{2cm} f(0) = \\
\lim_{x \to 2^-} f(x) &= \hspace{2cm} \lim_{x \to 2^+} f(x) &= \hspace{2cm} \lim_{x \to 2} f(x) &= \hspace{2cm} f(2) = 
\end{align*}
\]

List all the points of discontinuity of $f$ and classify them (as jump, removable, or infinite).
3. Let \( f(x) = 3x^2 - 5x \).
   
   a. Use the limit definition of the derivative to find the derivative of \( f \).
   
   b. Find the equation of the tangent line to \( f \) when \( x = 3 \).
   
   c. On what interval(s) is \( f \) decreasing?
(30 points)
4. Four pens will be built along a river by using 150 feet of fencing as shown in the diagram below. What dimensions $x$ and $y$ will maximize the total area of the four pens? Explain how you know your answer is a maximum.
5. Find the following by hand showing all work:

a. \( \frac{d}{dx} \int_0^{\sqrt{x}} e^{-t^2} \, dt \)

b. \( \lim_{\theta \to 0} \frac{1 - \cos \theta}{\sin(2\theta)} \)

c. \( \int (2 \sin x - e^x) \, dx \)

d. \( \int_{-1}^{2} (x^2 - x - 2) \, dx \)
For the function $f$ whose graph $y = f(x)$ is given below, sketch the graphs of the derivative $y = f'(x)$ and the antiderivative* $y = F(x)$ satisfying the additional condition that $F(-1) = 0$.

*Recall this means $F'(x) = f(x)$.
7. A ball is thrown straight up into the air with an initial velocity of 49 m/s. Accounting for wind resistance, its height above the ground is given by the equation $h(t) = 7350 - 245t - 7350e^{-t/25}$ meters (with $t$ measured in seconds).

a. Find the average speed of the ball for the first three seconds of its flight.

b. Find the exact speed of the ball 2 seconds after it was thrown.

c. Find the highest point reached by the ball.

d. Find the acceleration of the ball at its highest point.
8. The rate at which the world’s oil is being consumed is continuously increasing. Suppose the rate of
oil consumption (in billions of barrels per year) is given by the function \( r(t) \), where \( t \) is measured
in years and \( t = 0 \) is the start of 2004.

a. Write a definite integral which represents the total quantity of oil used between the start of
2004 and the start of 2014.

b. Suppose \( r(t) = 32e^{0.05t} \). Using a left-hand sum and a right hand sum with five subdivisions find
an approximate value for the total quantity of oil used between the start of 2004 and the start of
2014.

c. How many subintervals of the 10-year time span would you need in order to ensure an error
less than 0.05 billion barrels.