Chapters 3-4 Exam

Name (print): ______________________________

(30 points)
1. Find all of the critical points of \( y = (x - 1)^3 (x + 3)^2 \). Classify each one as a relative maximum, relative minimum, or neither and justify your answers.
2. Suppose $f$ is a function whose derivative is $f'(x) = \cos(x^2)$ and that $f(0) = -2$.
   a. Does $f$ have a critical point at $x = 0$? Fully justify your answer.
   b. Does $f$ have an inflection point at $x = 0$? Fully justify your answer.
   c. Find the equation of the tangent line to the graph of $f$ at $x = 0$.
   d. Use your answer to Part c to approximate $f(0.5)$.
   e. Use the formula, error $< \frac{M}{2} (x - x_0)^2$
      to find an error bound for your approximation in Part d.
(30 points)
3. The equation $3x^2 - 14xy + 7y^2 + 2y - 7xy = 24$ is an ellipse passing through the three points $(0, -2)$, $(6, 0)$, and $(12, 6)$. Find the equations of the tangent lines to this ellipse at two of these points, showing all of your work.
4. If you have 100 meters of fencing and you want to enclose a rectangular area up against a long, straight wall, what is the largest area you can enclose? Show all of your work and explain how you know that your answer is a maximum.
(30 points)
5. Find the following limits, showing all of your work.
   a. \( \lim_{x \to 1} \frac{x - 1}{\sqrt[3]{x + 7} - 2} \)
   b. \( \lim_{x \to 0} \frac{e^{2x} - 2x - 1}{x^2} \)
   c. \( \lim_{x \to 0} \frac{3x^2 + 7x - 4}{2x^2 - x + 5} \)