Ex 1

Name (print): ____________________________

(25 points)
1. The function $f$ graphed below is decreasing and has a removable discontinuity at $x = 2.5$.

![Graph of the function $f(x)$ with points at $(2.4, 4.31)$, $(2.5, 4.23)$, and $(2.6, y)$, and the equation $y = f(x)$.

a. Explain how you know that $\lim_{{x \to 2.5}} f(x)$ exists (without relying on the graph).

$f$ has a removable discontinuity at $x = 2.5$

This means $\lim_{{x \to 2.5}} f(x)$ exists.

b. What is an underestimate for $\lim_{{x \to 2.5}} f(x)$?

4.23

c. Write an algebraic expression for the error for this approximation.

$\left| 4.23 - \lim_{{x \to 2.5}} f(x) \right|

d. What is an error bound for this approximation?

4.31 - 4.23 = 0.08

e. Suppose $f$ is a rational function. What is one factor of the numerator? How do you know?

$(x - 2.5)$ must be a factor of the numerator.

We only get discontinuities in rational functions where the denominator is zero. The numerator must also be zero at this location in order for the limit to exist. $f(2.5) = 0 \iff (x - 2.5)$ is a factor.
(10 points)
2. Write the equation of a line through the point \((e^2, 3)\) with slope \(-\sqrt{2}\).

\[ y = 3 - \sqrt{2} (x - e^2) \]

(20 points)
3. According to the website www.RxMed.com, the elimination half-life of ibuprofen is approximately 2 hours. This means that every 2 hours, the amount of the pain-relief medication remaining active in the body is reduced by half. A typical dose for an adult is 400 milligrams (mg).

   a. Write a function giving the amount of ibuprofen remaining active in an adult as a function of time since a 400 mg dose is taken.

\[ A(t) = 400 \left(\frac{1}{2}\right)^{\frac{t}{2}} \]

   b. How long after taking a 400 mg dose is the amount of active ibuprofen in the body less than 1 mg? Show your work.

\[ \text{The amount of ibuprofen reaches } 1 \text{ mg when } \ln \frac{1}{400} = \frac{t}{2} \ln \left(\frac{1}{2}\right) \]

\[ t = 2 \frac{\ln 1400}{\ln \frac{1}{2}} = 17.2877 \]

   c. What is the continuous decay rate of ibuprofen in an adult body? Show your work.

\[ 400 \left(\frac{1}{2}\right)^{\frac{t}{2}} = 400 e^{kt} \]

\[ \left(\frac{1}{2}\right)^{\frac{t}{2}} = e^k \]

\[ k = \ln \left(\frac{1}{2}\right)^{\frac{1}{2}} = -0.3466 \]

   d. What is the instantaneous rate of decay of a 400 mg dose of ibuprofen when it is first taken? Show your work.

\[ \frac{dA}{dt} = kA = (-0.3466)(400) = -138.63 \text{ mg/hr} \]
(15 points)
4. Two angles, \( \alpha \) and \( \beta \), are marked on the circle of radius 1 centered at the origin shown below.

The coordinates of the point on the circle at angle \( \alpha \) are \((-0.8, 0.6)\).
The shortest arclength from the point on the circle at angle \( \beta \) to \((1, 0)\) is 1.9 units.

Find the following values

\[
\sin \alpha = \frac{0.6}{2}
\]
\[
\cos \alpha = \frac{-0.8}{2}
\]
\[
\beta = \frac{2\pi - 1.9}{2}
\]

(10 points)
5. Solve \( 5 \cdot e^x = 2^{x-1} \) for \( x \). Show your work.

\[
\ln (5e^x) = \ln (2^{x-1})
\]
\[
\ln 5 + \ln e^x = \ln (2^{x-1})
\]
\[
\ln 5 + x = (\ln 2)(x-1) = (\ln 2)x - \ln 2
\]
\[
\ln 5 + \ln 2 = (\ln 2 - 1) x
\]
\[
x = \frac{\ln 5 + \ln 2}{\ln 2 - 1}
\]
6. On June 21, the 172nd and longest day of the year, Greeley has 904 minutes of daylight. On December 21, the 355th and shortest day of the year, Greeley has 557 minutes of daylight. Write a sinusoidal function expressing the number of minutes of daylight as a function of the day of the year.

Use your function to determine the number of minutes of daylight in Greeley on September 20, the 263rd day of the year.

\[
\text{Amplitude} = \frac{904 - 557}{2} = 173.5
\]

\[
\text{Period} = 365
\]

\[
\text{Vertical shift} = \frac{904 + 557}{2} = 730.5
\]

\[
\text{Horizontal shift (for cosine)} = 172
\]

\[
y = 730.5 + 173.5 \cos \left( \frac{2\pi}{365} (t - 172) \right)
\]

(using sine, the spring equinox is on March 20, the 79th day of the year.

\[
y = 730.5 + 173.5 \sin \left( \frac{2\pi}{365} (t - 79) \right)
\]

\[
y(263) = 731
\]

There will be 731 minutes of daylight on September 20.