Problem 1. A skydiver jumps from an airplane and her velocity is then $v(t)$ ft/s after $t$ seconds of freefall in a tucked position, so that she continually speeds up.

a. Use an integral to express the distance the skydiver has fallen after 30 seconds of freefall. (Assume that distance is measured from the jump altitude so that the velocity $v(t)$ is positive.)

b. Given the following data, find both an underestimate and an overestimate for the distance the skydiver has fallen after 30 seconds of freefall.

<table>
<thead>
<tr>
<th>time $t$ (seconds)</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>velocity $v(t)$ (ft/s)</td>
<td>0</td>
<td>160</td>
<td>192</td>
<td>198</td>
</tr>
</tbody>
</table>

c. If speed data were available every 2 seconds, what would be the difference between the underestimate and overestimate based on 15 subdivisions of the time interval?

d. How many subintervals would be needed to have the underestimate and overestimate accurate to within 10 ft?

Problem 2. True or False?

a. If $f(x) > 0$, then $\int_a^b f(x) \, dx$ measures the area under the graph of $f$ between $x = a$ and $x = b$.

b. If $F'(x) = f(x)$ is continuous, then $\int_a^b f(x) \, dx$ measures the total change in $F$ between $x = a$ and $x = b$.

Problem 3. Suppose $C(t)$ is the power, in kJ/h, produced by a solar panel $t$ hours after sunrise on a typical summer day. Give practical interpretations of

$$\int_0^{27} C(t) \, dt = 270$$

and

$$\frac{1}{12} \int_0^{12} C(t) \, dt = 288.$$

Problem 4. The table below gives the expected growth rate, $g(t)$, in ounces per week, of the weight of a baby in its first 54 weeks of life (which is slightly more than a year). Assume for this problem that $g(t)$ is a decreasing function.

<table>
<thead>
<tr>
<th>week $t$</th>
<th>0</th>
<th>9</th>
<th>18</th>
<th>27</th>
<th>36</th>
<th>45</th>
<th>54</th>
</tr>
</thead>
<tbody>
<tr>
<td>growth rate $g(t)$</td>
<td>6</td>
<td>6</td>
<td>4.5</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

a. Using six subdivisions, find an overestimate and underestimate for the total weight gained by a baby over its first 54 weeks of life.

b. How frequently over the 54 week period would you need the data for $g(t)$ to be measured to find overestimates and underestimates for the total weight gain over this time period that differ by 0.5 lb (8 oz)?

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Problem 5.

a. Using 4 equal subdivisions, find a Riemann sum which is an underestimate for

\[ \int_2^4 \ln(x) \, dx . \]

Sketch a graphical representation of your Riemann sum on the axes below, and write “LHS” or “RHS” next to your figure to indicate whether you are using a left-hand sum or a right-hand sum. Write out the terms of the Riemann sum using exact values (no calculator approximations). There is no need to simplify the sum.

b. Show that \( F(x) = x \ln(x) - x + C \) is an antiderivative for \( \ln(x) \), where \( C \) is any constant. In other words, show \( F'(x) = \ln(x) \).

c. Using Part b, find the exact value of the integral \( \int_2^4 \ln(x) \, dx \).

Problem 6. The rate at which the world’s oil is being consumed is continuously increasing. Suppose the rate of oil consumption (in billions of barrels per year) is given by the function \( r(t) \), where \( t \) is measured in years and \( t = 0 \) is the start of 2004.

a. Write a definite integral which represents the total quantity of oil used between the start of 2004 and the start of 2014.

b. Suppose \( r(t) = 32e^{0.05t} \). Using a left-hand sum and a right hand sum with five subdivisions find an approximate value for the total quantity of oil used between the start of 2004 and the start of 2014.

c. Find an error bound for your estimates in part (b).

d. Give an algebraic expression for the error bound for any number of rectangles.

e. How many subintervals of the 10-year time span would you need in order to ensure an error less than 0.05 billion barrels.

f. Interpret each of the five terms in the left-hand sum from part (b) in terms of oil consumption.

Problem 7. The NASA Q36 Robotic Lunar Rover can travel up to 3 hours on a single charge and after \( t \) hours of traveling, its speed is \( v(t) \) miles per hour given by the function \( v(t) = \sin \sqrt{9 - t^2} \).

a. Write an integral that expresses the distance traveled by the Q36 during its first two hours of operation.

b. How many terms would be required in a Riemann sum for your integral in part a to be accurate to within one foot, i.e., 1/5280 miles?
Problem 8. Use your calculator to draw a very accurate graph of $w(s) = s^3$ on the interval $(0,3]$.

a. Explain how to use your graph (not a calculator at this point) to estimate the value of $\int_0^3 s^3 \, ds$. Try to get close to the correct answer, but your explanation is the most important part of your answer.

b. Now use your calculator to evaluate a Riemann sum that is an underestimate but within 0.5 of the actual value of $\int_0^3 s^3 \, ds$. Explain how you know your sum is an underestimate and that it is sufficiently accurate.

Problem 9. Find the average value of $f(x) = \frac{1}{x}$ between $x=1$ and $x=10$.

Problem 10. Let $f$ and $g$ be continuous functions for all real numbers.

a. If $\int_2^5 (2f(x) + 3) \, dx = 17$, find $\int_2^5 f(x) \, dx$.

b. If $f$ is odd and $\int_{-3}^3 f(x) \, dx = 30$, find $\int_{-3}^3 f(x) \, dx$.

c. If $f$ is even and $\int_{-2}^2 (f(x) - 3) \, dx = 8$, find $\int_{-2}^2 f(x) \, dx$.

d. If the average value of $f$ on the interval $2 \leq x \leq 5$ is 4, find $\int_2^5 (3f(x) + 2) \, dx$. 