

1	2
3	4

The Set - Legal Moves

- I The penny stays stationary.
- H The penny moves horizontally such that it remains in the grid. (\leftrightarrow)
- V The penny moves vertically such that it remains in the grid. (\updownarrow)
- D The penny moves diagonally such that it remains in the grid. (\nearrow or \nwarrow)

Operation - Composition (\circ)

$H \circ V$ means: do the H move followed by the V move.

Task 1A

Does the answer to $H \circ V$ depend on which square the penny starts in? Show how you determined your answer.

Task 1B

Does the answer to $D \circ V$ depend on which square the penny starts in? Show how you determined your answer.

Task 1C

See if any other composition pairs depend upon which square the penny starts in. What can you conclude about the need to know which square the penny starts in?

Task 1D

Why do you think your conclusion in Task 1C is true?

Task 2A

To determine if this set of moves is closed under composition, we can create a table to show the results of every possible composition pair. One square of the table has been filled in for you: $H \circ V = D$. For every square in the table, the move on the left hand column is the first move, and the move above (i.e., the top row) is the second move.

1	2
3	4

○	I	H	V	D
I				
H			D	
V				
D				

Task 2B

Are the penny moves closed under the operation of composition? Explain how the table you created in Task 2A supports your answer.

Task 2C

What other observations can you make about the penny moves under composition? Hint: you might look for patterns in the table.

Task 3A

Put the penny in square 1. Then do the following moves and write down the square the penny ends up in. What single move is the same as all the moves you did?

1	2
3	4

D◦D◦V◦H◦I◦D◦H

Task 3B

Repeat Task 3A, but start the penny in different squares. Does it matter what square you start in?

Task 3C

Is there any sequence of H, D, I, and V moves that does not result in a single move of H, D, I, or V? How did you determine your answer?

Task 3D

How is closure related to your answer in Task 3C?

Task 4A

List two things you learned about mathematics by doing this worksheet.

Task 4B

List any questions you have.