University of Northern Colorado

Educational Mathematics PhD Comprehensive Exam

Summer 2013, Part 1

Respond to **TWO** of the following three numbered questions. This part of the exam focuses on general knowledge, mainly from coursework and readings in MED 610, 700, 701 and 702.

Provide responses to entire questions. All responses should be in essay form; aim for clarity and explicitness, as well as thoroughness, concision, and coherence in your writing. Define all technical terminology that you use in your responses and be explicit about how you are applying the ideas.

Adhere to the following formatting guidelines:

- Write your name on the first page of each response.
- Start each answer on a new page and double space the final version.
- If you use diagrams in your responses, label each with a title (e.g., "Figure x") and insert a clear reference to each one in the appropriate place in your narrative. Figures may be hand-drawn and submitted with the electronic copy of the exam.
- In all cases, support assertions with citations from the literature, as appropriate, using APA format: (Author(s), Year).
- **SAVE** your work **OFTEN** and then save your final copy on the USB drive provided by the proctor.

1. Describe the role of theory in mathematics education research. Compare and contrast two learning theories that have been used in mathematics education research with respect to their epistemological stance, the meaning and process of learning, factors that influence learning, and the implications for teaching and research. Discuss limitations and critiques of each theory.

2. In a meta-analysis, Ma (1999) found the relationship between anxiety and mathematics achievement was consistently negative across three grade bands, 4–6, 7–9, and 10–12.

> **Abstract:** In this meta-analysis I examined 26 studies on the relationship between anxiety toward mathematics and achievement in mathematics among elementary and secondary students. The common population correlation for the relationship is significant (–.27). A series of general linear models indicated that the relationship is consistent across gender groups, grade-level groups, ethnic groups, instruments measuring anxiety, and years of publication. The relationship, however, differs significantly among instruments measuring achievement as well as among types of publication. Researchers using standardized achievement tests tend to report a relationship of significantly smaller magnitude than researchers using mathematics teachers' grades and

researcher-made achievement tests. Published studies tend to indicate a significantly smaller magnitude of the relationship than unpublished studies. There are no significant interaction effects among key variables such as gender, grade, and ethnicity.

Although this relation is generally negative, how anxiety impacts individual students' mathematics achievement remains a potential topic of further research. Describe and justify the design of a qualitative study that addresses some aspect of the nuanced interaction between anxiety and mathematics achievement. As part of your justification, identify at least two articles on which to base your study and briefly explain how each article informs your research design choices.

Ma, X. (1999). A meta-analysis of the relationship between anxiety towards mathematics and achievement in mathematics. *Journal for Research in Mathematics Education*, *30*, 520-540.

3. Discuss strengths and weakness of quantitative and qualitative research methodologies and how the mixed methods research paradigm attempts to leverage the productive aspects of each. Provide two examples of possible mixed method research studies, one in which qualitative methods are followed by quantitative methods, and second in which quantitative methods are followed by qualitative methods. Your examples should illustrate the navigation of critical issues in mixed methodology such as the alignment of philosophical positions, rationale, data collection and analysis, mixing and integration of procedures, and validation strategies. Draw on relevant literature to support your arguments and examples.

University of Northern Colorado

Educational Mathematics PhD Comprehensive Exam

Summer 2013, Part 2

Respond to **TWO** of the following three numbered questions. This part of the exam focuses on the short reading list of articles and book chapters specified at the beginning of the summer and loaded onto your USB drive. You may also access ***unmarked*** copies of the items on the short reading list during the exam. Your responses should be framed by a comprehensive treatment of the relevant ideas contained in the specified articles and book chapters. You should also draw on additional literature to develop broader support for your answers. Any student who uses only the reading list items in their comparisons and discussions will not pass the exam.

Provide responses to entire questions. All responses should be in essay form; aim for clarity and explicitness, as well as thoroughness, concision, and coherence in your writing. Define all technical terminology that you use in your responses and be explicit about how you are applying the ideas.

Adhere to the following formatting guidelines:

- Write your name on the first page of each response.
- Start each answer on a new page and double space the final version.
- If you use diagrams in your responses, label each with a title (e.g., "Figure x") and insert a clear reference to each one in the appropriate place in your narrative. Figures may be hand-drawn and submitted with the electronic copy of the exam.
- In all cases, support assertions with citations from the literature, as appropriate, using APA format: (Author(s), Year).
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1. Tall and Vinner (1981) describe students' concept images related to limits and continuity and their interaction with concept definitions and potential conflict factors. Sfard (1992) characterizes an "ontological duality" between structural and operational approaches to the development of the function concept. Choose a mathematical concept (other than limit or function) and apply these two frameworks to:

   a. Give an example of a personal concept that differs from a standard concept definition and identify potential conflict factors.

   b. Describe a method of instruction regarding your chosen concept adhering to the principles Sfard infers from her work and detailing the intended processes of *interiorization*, *condensation*, and *reification*.

   Discuss how these two perspectives inform one another, conflict, or leverage different aspects of student learning.

2. Analyze Cory & Garofalo's (2011) use of interactive sketches in an instructional sequence on the formal definition of sequence convergence in terms of Gravemeijer, Cobb, Bowers, & Whitenack's (2000) distinction between exploratory and expressive approaches to developing students' models. Draw from your analysis to redesign the instructional sequence using principles of Realistic Mathematics Education (RME). In particular, articulate a clear learning goal consistent with both Cory & Garofalo's original goals and the perspective of RME and articulate an associated hypothetical learning trajectory. Attend to and reconcile the two, often competing, goals of leveraging the "the reflexive relation between symbolizing and sense making" (p. 235) and "enabling students to reason powerfully with conventional symbolizations" (p. 236). Identify expected aspects of horizontal and vertical mathematization in your learning trajectory and clearly describe the role of the teacher.

3. Briefly describe and compare the views of mathematical reasoning categories introduced by authors Simon (1996) and Lithner (2008). In particular discuss how transformational reasoning described by Simon (1996) could be viewed using Lithner's (2008) imitative and creative reasoning framework. In addition, discuss possible ways that each reasoning category could inform reflexive (low road) transfer and mindful (high road) transfer mechanisms described by Perkins and Salomon (1992).

References

Cory, B. L., & Garofalo, J. (2011). Using dynamic sketches to enhance preservice secondary mathematics teachers' understanding of limits of sequences, *Journal for Research in Mathematics Education*, *42*(1), 65-97.

Gravemeijer, K., Cobb, P., Bowers, J. and Whitenack, J. (2000). Symbolizing, Modeling, and Instructional Design. In Paul Cobb, Erna Yackel, & Kay McClain (Eds.) *Symbolizing and Communicating in Mathematics Classrooms: Perspectives on Discourse, Tools, and Instructional Design.* (pp. 225-273). Mahwah, NJ: Erlbaum and Associates.

Lithner, J. (2008). A research framework for creative and imitative reasoning. *Educational Studies in Mathematics*, *67*(3), 255-276.

Perkins, D. N., & Salomon, G. (1992). Transfer of learning. *International Encyclopedia of Education*, Second Edition. Oxford: Pergamon Press. [Online]. http://learnweb.harvard.edu/alps/thinking/docs/traencyn.htm.

Sfard, A. (1992). Operational origins of mathematical objects and the quandary of reification – The case of function. In G. Harel & E. Dubinsky (Eds.), *The Concept of Function: Aspects of Epistemology and Pedagogy* (MAA Notes, Vol. 25, pp. 59-84). Washington, DC: Mathematical Association of America.

Simon, M. (1996). Beyond Inductive and Deductive Reasoning: The Search for a Sense of Knowing. *Educational Studies in Mathematics, 30*(2), 197-210.

Tall, D., & Vinner, S. (1981). Concept image and concept definition in mathematics with particular reference to limits and continuity. *Educational studies in mathematics, 12*(2), 151-16.

MED Comps: Summer 2012

PART I

ANSWER ALL (3 hours with computer). This part of the exam focuses on general knowledge, mainly from coursework and readings in MED 610, 700, 701 and 702.

Provide responses to entire questions. All responses should be in essay form; aim for clarity and explicitness, as well as thoroughness, concision, and coherence in your writing. Define all technical terminology that you use in your responses and be explicit about how you are applying the ideas.

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1. The following is the abstract for Lai, Weber, and Majía-Ramos (2012):

   > In this article, we report two studies investigating what mathematicians value in a pedagogical proof. Study 1 is a qualitative study of how eight mathematicians revised two proofs that would be presented in a course for mathematics majors. These mathematicians thought that introductory and concluding sentences should be included in the proofs, main ideas should be formatted to emphasize their importance, and extraneous or redundant information should be removed to avoid distracting or confusing the reader. Study 2 is a quantitative study assessing the extent to which a larger group of mathematicians ($N = 110$) agreed or disagreed with the eight mathematicians interviewed in Study 1. This quantitative study confirmed the findings of Study 1 by demonstrating a high degree of agreement among mathematicians regarding how they would revise proofs for pedagogical purposes.

   a. Based only on the Lai et al. (2012) article abstract, provide a brief critique for basing Study 2 on Study 1. Specifically, explain the advantages and disadvantages of following the qualitative study with a quantitative study.

   b. Use criteria for rigorous quantitative research as described in Gall, Gall, and Borg (2006) and/or other relevant sources to describe characteristics of the research that you would expect Lai et al. (2012) to have considered in Study 2.

Lai, Y., Webber, K., Majía-Ramos, J. P. (2012). Mathematicians' perspectives on features of a good pedagogical proof. *Cognition and Instruction, 30*(2), 146–169

Gall, M. D., Gall, J. P., & Borg, W. R. (2012) *Educational Research: An Introduction* (8th ed.). Boston: Allyn and Bacon.

2. Choose an area of mathematics education research and synthesize the findings of at least three different published research articles related to this area. Describe an important theoretical perspective specifying assumptions about the nature of knowledge and learning that has been employed to conduct research in this area and describe its role and impact in the research that you discussed.

3. You have recently been hired as a post-doc researcher to help evaluate a newly designed inquiry-based learning (IBL) college algebra course for first year college students. The course was designed to cover typical college algebra topics such as solving equations, functions, exponential and logarithmic functions, etc. through IBL. The developer of the course summarized the important aspects of IBL incorporated into the course as:

> "Like 'discovery learning' (Bruner, 1961; Dewey, 1938), problem-based learning, and other "inductive teaching" approaches (Prince & Felder, 2007), IBL invites students to work out ill-structured but meaningful challenges. By drawing on their own experience and prior knowledge, exploring their environment or performing experiments, and wrestling with questions and controversies, students learn what they need to know to address the challenge. In this process, students build critical thinking, analysis, and communication skills and learn to use resources efficiently. IBL is also a form of "active learning," the goal of which is to engage students in the learning process, and thereby activate responsibility for their own learning processes" (Laursen et.al., 2011, p. 34).

You were assigned to design a qualitative study that contributes to the evaluation of Laursen's (2011) stated claims and goals in this newly designed course. Provide an overview of your evaluation proposal. As part of this overview address major issues of educational research, including the research question(s), relevant literature, a description of the setting and participants, the proposed method(s) of data collection and analysis methods, including ways to ensure valid and reliable results, and the limitations of your proposed study.

MED Comps: Summer 2012
PART II

ANSWER TWO (3 hours with computer). Respond to TWO of the three items. This part of the exam focuses on the short reading list of articles and book chapters specified at the beginning of the summer and loaded onto your USB drive. You may also access ***unmarked*** copies of the items on the short reading list during the exam. Your responses should be framed by a comprehensive treatment of the relevant ideas contained in the specified articles and book chapters. You should also draw on additional literature to develop broader support for your answers. Any student who uses only the reading list items in their comparisons and discussions will not pass the exam.

Provide responses to entire questions. All responses should be in essay form; aim for clarity and explicitness, as well as thoroughness, concision, and coherence in your writing. Define all technical terminology that you use in your responses and be explicit about how you are applying the ideas.

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1. von Glassersfeld characterized *accommodation,* one of the major learning processes of a Piagetian constructivist perspective in general terms as follows:

   > [C]ognitive change and learning in a specific direction takes place when a scheme, instead of producing the expected result, leads to a perturbation, and perturbation, in turn, to an accommodation that maintains or re-establishes equilibrium. (p. 68)

   Elaborate the meaning of this statement from a radical-constructivist perspective and explain the relationship of accommodation to reflective abstraction. Contrast your response to the characterizations of *generalization* provided by Ellis (2011) and *the ways of thinking* discussed by Harel and Sowder (2005).

2. Prawat & Floden (1994) contrast three different underlying world views and discuss a general learning theory consistent with each viewpoint. Describe three published mathematics education research studies (different from those discussed by Prawat & Floden) that were conducted from the three different perspectives, and justify your exemplifications by making explicit connections between the characteristics of those studies and the descriptions provided by Prawat & Floden.

3. Clement (2000) described educational research as ranging from generative to convergent. Briefly describe what Clement meant by this characterization and discuss the role that each type of research plays in the general endeavor of mathematics education research. Categorize Wilkins & Norton's (2011) research according to Clement's distinctions and clearly justify your claims. Similarly, using Clement's framework, categorize the important literature on which Wilkins & Norton built their research. Finally, describe how both the previous researchers and Wilkins & Norton have each contributed to developing theory about children's reasoning about fractions.

Mathematics Education Doctoral Comprehensive Exam Reading List
Summer 2012

Clement, J. (2000). Analysis of clinical interviews: Foundations and model viability. In A. Kelly & R. Lesh (Eds.), *Handbook of research design in mathematics and science education.* Mahweh, NJ: Lawrence Erlbaum Associates, 547-589.

Ellis, A. B. (2011). Generalizing-promoting actions: How classroom collaborations can support students' mathematical generalizations. *Journal for Research in Mathematics Education, 42*(4), 308-345.

Harel, G., & Sowder, L. (2005). Advanced mathematical-thinking at any age: Its nature and its development. *Mathematical Thinking and Learning, 7*(1), 27-50.

Prawat, R. & Floden, R. (1994). Philosophical perspectives on constructivist views of learning. *Educational Psychology, 29,* 37-48.

Von Glasersfeld, E. (1995). *Radical constructivism: A way of knowing and learning.* London: Falmer Press. (Chapter 3. Piaget's Constructivist Theory of Knowing, pp. 53-75 and Chapter 5. Reflection and Abstraction, pp. 89-112.)

Wilkins, J. L. M., & Norton, A. (2011). The splitting loope. *Journal for Research in Mathematics Education, 42*(4), 386-416.