Testing Claims about Differences between Two Means – Dependent Samples

Step 0: Verify Assumptions
The hypothesis test of the difference of two means involving dependent samples has three assumptions.
1. The samples are obtained using simple random sampling.
2. The sample data are matched pairs.
3. The differences include no outliers and are normally distributed or the sample size, \( n \), is large (\( n \geq 30 \)).

Step 1: State the Hypothesis
A claim is made regarding the difference between two means from matched-paired data. This claim is used to determine the null and alternative hypotheses. The hypotheses can be structured in one of the following ways:

<table>
<thead>
<tr>
<th>Two-Tailed</th>
<th>Left-Tailed</th>
<th>Right-Tailed</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H_0: \mu_d = d_0 )</td>
<td>( H_0: \mu_d \geq d_0 )</td>
<td>( H_0: \mu_d \leq d_0 )</td>
</tr>
<tr>
<td>( H_1: \mu_d \neq d_0 )</td>
<td>( H_1: \mu_d &lt; d_0 )</td>
<td>( H_1: \mu_d &gt; d_0 )</td>
</tr>
</tbody>
</table>

Step 2: Select a Level of Significance
The selection of the level of significance \( \alpha \) is done based on the seriousness of making a Type I error. (The typical value of \( \alpha \) is 0.05.)

Step 3: Calculate the Test Statistic
The test statistic represents the number of standard deviations the mean difference \( \bar{d} \) is from the claimed population difference, \( d_0 \), based on the standard deviation of the differences, \( s_d \) and the sample size, \( n \). The test statistics approximately follows Student’s \( t \)-distribution with \( n - 1 \) degrees of freedom.

\[
t_0 = \frac{\bar{d} - d_0}{\frac{s_d}{\sqrt{n}}}
\]
Step 4: Determine the Decision Criterion

The Classical Approach: Find the Critical Value
The level of significance is used to determine the critical value, represented by the \( t \)-values in the figures below based on the degrees of freedom, \( n - 1 \). The critical region includes the values of the shaded region. The shaded region has area \( \alpha \).

The Modern Approach: Find the \( p \)-Value
Based on the critical value \( t_0 \), determine the probability that a sample mean is further from the mean than is hypothesized using the \( t \)-distribution with \( n - 1 \) degrees of freedom. This is represented by the shaded region in the figures below.

Step 5: Make a Decision
Reject the null hypothesis if the test statistic lies in the critical region or the probability associated with the test statistic is less than the level of significance.
Do not reject the null hypothesis if the test statistic does not lie in the critical region or the probability associated with the test statistic is greater than or equal to the level of significance.

Step 6: State the Conclusion
State the conclusion of the hypothesis test based on the decision made and with respect to the original claim.

<table>
<thead>
<tr>
<th>Original Claim is ( H_0 )</th>
<th>Original Claim is ( H_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reject ( H_0 )</td>
<td>There is sufficient evidence (at the ( \alpha ) level) to reject the claim that \ldots</td>
</tr>
<tr>
<td>Do Not Reject ( H_0 )</td>
<td>There is not sufficient evidence (at the ( \alpha ) level) to reject the claim that \ldots</td>
</tr>
</tbody>
</table>