Distributions of Sample Proportions

The goal of this activity is to use a graphics calculator to investigate the distributions of sample proportions through a simulation.

The following simulation involves a distribution called the chi-squared distribution.

**The Sample Proportion**

Suppose a simple random sample of size $n$ is obtained from a population in which each individual either does or does not have a certain characteristic. The **sample proportion** is given by

$$
\hat{p} = \frac{x}{n}
$$

where $x$ is the number of individuals in the sample with the specified characteristic. The sample proportion is a statistic that estimates the population proportion, $p$.

According to the M&M’S® website (us.mms.com), the distribution of blue candies in bags of Peanut M&Ms is 23%. The 1.74 oz. bags of Peanut M&Ms typically contain about 20 candies. Simulate obtaining 100 bags of Peanut M&Ms to describe the distribution of blue candies.

1. To simulate the number of blue Peanut M&M in a 1.74 oz. bag, enter the following command:

   `randBin(20,.23)`

   What value did you get?

2. To calculate the sample proportion of blue M&M in a bag containing 20 candies, enter the following command:

   `randBin(20,.23)/20`

   What value did you get? What is the proportion of other colors in the sample?

3. To simulate all 100 the sample proportions, enter the following command (this will take a minute):

   `seq(randBin(20,.23)/20,X,1,100,1)`

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4. Create a histogram in STAT PLOT of the sample proportions using the viewing window [0,1,.05]. Sketch the graph to the right.

5. Describe the sample proportion distribution.

6. Verify that the distribution of sample proportions of the simulation approximates the mean and standard deviation of the sampling distribution of \( \hat{p} \).

7. A recent study involving 1,506 randomly selected adults via telephone interviews found that 1130 reported having had at least one symptom of a sleep problem a few nights a week or more within the past year. In a random sample of 110 adults, determine the probability that at least 90 report having had at least one symptom of a sleep problem a few nights a week or more within the past year.

**Sampling Distribution of \( \hat{p} \)**

For a simple random sample of size \( n \) such that \( n \leq N \) (i.e., the sample size is no more than 5% or the population size)
- The shape of the sampling distribution of \( \hat{p} \) is approximately normal provided \( npq \geq 10 \)
- The mean of the sampling distribution of \( \hat{p} \) is \( \mu_\hat{p} = \hat{p} \)
- The standard deviation of the sampling distribution of \( \hat{p} \) is

\[
\sigma_\hat{p} = \sqrt{\frac{pq}{n}}
\]