Kepler’s Law

The goal of this activity is to use logarithmic transformations to linearize power relations.

The time it takes to complete its orbit around the sun is called the planet’s sidereal year. In 1618, Johann Kepler discovered that the sidereal year of a planet is related to the distance from the sun. The following data show the distances that the planets are from the sun (in astronomical units, AU) and their sidereal years (in Earth years).

<table>
<thead>
<tr>
<th>Planet</th>
<th>Mercury</th>
<th>Venus</th>
<th>Earth</th>
<th>Mars</th>
<th>Jupiter</th>
<th>Saturn</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance from Sun ($d$)</td>
<td>0.39</td>
<td>0.72</td>
<td>1.00</td>
<td>1.53</td>
<td>5.19</td>
<td>9.54</td>
</tr>
<tr>
<td>Sidereal Year ($y$)</td>
<td>0.24</td>
<td>0.62</td>
<td>1.00</td>
<td>1.88</td>
<td>11.9</td>
<td>29.5</td>
</tr>
</tbody>
</table>

1. Enter the data above in your calculator. Enter the distance in L1 and the sidereal year in L2. Be sure to enter them in order so that the times correspond to each other.

2. Create a scatter plot of the data. Construct the scatter diagram in the graph below.

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1 Uranus was discovered on March 13, 1781, Neptune on September 23, 1846; Pluto is no longer considered a planet but rather a dwarf planet by the International Astronomical Union (IAU).
3. Briefly describe any visible pattern between the weekend and the earnings.

4. Calculate the logarithm of the sidereal year in L4. Record the data in the table.

<table>
<thead>
<tr>
<th>Distance from Sun (d)</th>
<th>0.39</th>
<th>0.72</th>
<th>1.00</th>
<th>1.53</th>
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<td>29.5</td>
</tr>
<tr>
<td>( Y = \log y )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5. Create a scatter plot of the distance L1 and the logarithm of sidereal year L4. Construct the scatter diagram in the graph below.

![Scatter Diagram](image)

6. Find the least-squares regression line of the transformed data.

\[ Y = \text{_______} \times d + \text{_______} \]

7. Explain whether this regression line is a good model of the transformed data.
8. Calculate the logarithm of the distance in \( L_3 \). Record the data in the table.

\[
\begin{array}{cccccc}
D &=& \log d \\
\text{Distance from Sun (}d\text{)} & 0.39 & 0.72 & 1.00 & 1.53 & 5.19 & 9.54 \\
\text{Sidereal Year (}y\text{)} & 0.24 & 0.62 & 1.00 & 1.88 & 11.9 & 29.5 \\
\end{array}
\]

9. Create a scatter plot of the logarithm of distance \( L_3 \) and the logarithm of sidereal year \( L_4 \).

Conduct the scatter diagram in the graph below.

10. Find the least-squares regression line of the transformed data using \( L_3 \) and \( L_4 \).

\[ Y = _______ \times D + _______ \]

11. Explain whether this regression line is a good model of the transformed data.

12. Use properties of logarithms to find exponential equation of best fit for the original data based on the least-squares regression line in part 9. Graph it on the scatter plot with the original data.
13. Use the power equation of best fit to estimate the following.

a. Uranus’ distance from the sun is 19.19AU. Estimate the sidereal year of Uranus.

b. The sidereal year of Neptune is 165 years. Estimate the distance from the sun of Neptune.

c. Does the model apply to the following dwarf planets in the solar system? Explain.

<table>
<thead>
<tr>
<th>Dwarf Planet</th>
<th>Ceres</th>
<th>Pluto</th>
<th>Eris</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance from Sun (d)</td>
<td>2.76</td>
<td>39.48</td>
<td>67.67</td>
</tr>
<tr>
<td>Sidereal Year (y)</td>
<td>4.60</td>
<td>248.0</td>
<td>~557</td>
</tr>
</tbody>
</table>

Source: en.wikipedia.org

d. Makemake (pronounced MAH-kay MAH-kay) was discovered on March 31, 2005, and designated a dwarf planet in July 2008. It is currently 52 AU from the Sun and has a sidereal year of nearly 309.88 years (en.wikipedia.org). Does the model apply to this information about Makemake? Explain.

13. Perform a Power Regression using your calculator on the original data. How does the compare with the power equation in part 12?