Le Poisson, Le Poisson

The goal of this activity is to understand the Poisson probability distribution.

A park ranger monitors 100 people fishing in a lake from 6 a.m. to 9 a.m. Based on their locations, each fisher has an equal probability of catching a fish per unit time. The ranger collected the following information.

<table>
<thead>
<tr>
<th>No. of fish caught</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of Fishers</td>
<td>10</td>
<td>23</td>
<td>27</td>
<td>20</td>
<td>12</td>
<td>5</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>100</td>
</tr>
</tbody>
</table>

1. Determine the mean and standard deviation of these data.

2. Explain why the lake fishing example satisfies the Poisson process.

3. Determine the probability that a fisher will catch exactly 3 fish in the three hour period. (Note: \( \lambda t \) equals the mean of the distribution.) Compare this answer to the proportion provided by the data.

**The Poisson Process**

A random variable \( X \), which is the number of success in a fixed interval, follows a Poisson process provided the following conditions are met.

1. The probability of two or more successes in any sufficiently small subinterval is 0.
2. The probability of success is the same for any two intervals of equal length.
3. The number of successes in any interval is independent of the number of successes in any other interval provided the intervals are not overlapping.

**Poisson Probability Distribution Function**

If \( X \) is the number of successes in an interval of fixed length, \( t \), the probability formula for \( X \) is

\[
P(x) = \frac{(\lambda t)^x}{x!} e^{-\lambda t}, \quad x = 0, 1, 2, 3, \ldots
\]

where \( \lambda \) represents the average number of occurrences of the event in some interval of length 1.
A Web site manager determined that the number of hits to the site is consistently two per minute during the evening hours of 7 p.m. to 9 p.m. Let the number of hits to the Web site be the random variable $X$. Explain why this is a Poisson process.

4. A system upgrade takes five minutes to install, a time when the server is down.

   a. Determine a formula for $P(x)$.

   b. Determine and interpret the probability that exactly 8 users receive a message that the server is down.

   d. Determine the mean and standard deviation of the number of hits the server will have while it is down.

Mean (or Expected Value) and Standard Deviation of a Poisson Random Variable

A random variable $X$ that follows a Poisson process with parameter $\lambda$ has mean (or expected value) and standard deviation given by the formulas

$$
\mu_x = \lambda \cdot t \quad \text{and} \quad \sigma_x = \sqrt{\lambda \cdot t} = \sqrt{\mu_x}
$$

where $t$ is the length of the interval.

Poisson Probability Distribution Function

Based on the mean given above, an alternative formula for the Poisson probability distribution is:

$$
P(x) = \frac{\mu^x}{x!} e^{-\mu}, \quad x = 0, 1, 2, 3, \ldots
$$

where $\mu$ is the mean of the Poisson random variable.
5. The student copy machine in the library requires a maintenance call an average of two times per month. Assume the number of required maintenance calls follows a Poisson distribution. Determine the probability that the copy machine would require more than 4 maintenance calls in any given month.

The TI graphing calculator provides a built-in function for determining both the Poisson probability distribution function and the cumulative distribution function. They are located in the DISTR menu above the VARS key. The syntax of the functions are given below.

\[ \text{poissonpdf}(\mu, \text{value}), \text{which results in } P(x) \]
\[ \text{poissoncdf}(\mu, \text{value}), \text{which results in } P(x \leq \text{value}) \]

6. In July 2006, Denver initiated a new 3-1-1 service as a way for citizens to get non-emergency information. The system takes calls seven days per week from 6 a.m. to 11 p.m. According to news reports (www.denvergov.org), the anticipated rate of calls is 130 calls per hour. Assume the number of calls received by the 3-1-1 service follows a Poisson distribution. Determine the probability that fewer than 120 calls will be received in an hour.