Normal Approximation to the Binomial Probability Distribution

The goal of this activity is to understand how the normal probability distribution may be used to approximate the binomial probability distribution.

State the four criteria for a binomial probability experiment.

Clinical trials of the popular allergy medicine Allegra-D found that 13% of the trial subjects reported headache as an adverse reaction to the drug. A follow-up study involves a random sample of 2150 allergy sufferers. What is the probability that less than 250 report headache?

To answer this question by hand would involve 250 calculations!

\[ P(X < 200) = P(0) + P(1) + P(2) + \ldots + P(249) \]

Statisticians have found a simpler way. The figure below presents a binomial distribution overlaid with the normal probability distribution with the same mean and standard deviation:

\[ \mu = n \cdot p \quad \text{and} \quad \sigma = \sqrt{n \cdot p \cdot (1 - p)} \]

As suggested by the graph and under certain conditions, the normal probability distribution may be used to approximate the binomial probability distribution.
Consider the binomial random variable $X$ with $n = 45$ and $p = 0.40$. The figure below shows a histogram of the probability distribution with a normal curve with $\mu_X = 45 \cdot 0.40 = 18$ and $\sigma_X = \sqrt{45 \cdot 0.40 \cdot 0.60} = \sqrt{10.8}$.

We know that the area of the rectangle corresponding to $X = 16$ represents $P(16)$ of the binomial probability distribution. The width of the rectangle is 1, so it extends from $X = 15.5$ to $X = 16.5$. Notice that the area under the normal curve from $X = 15.5$ to $X = 16.5$ is approximately equal to the area of the rectangle corresponding to the value of $X = 16$. Therefore, the area under the normal curve between $X = 15.5$ to $X = 16.5$ is approximately equal to $P(16)$.

The process of adding and subtracting 0.5 from the value of the binomial random variable when using the normal probability distribution is called the **correction for continuity**.
Continue to consider the binomial random variable \( X \) with \( n = 45 \) and \( p = 0.40 \). The mean and standard deviation of \( X \) are \( \mu_X = 18 \) and \( \sigma_X = \sqrt{10.8} \), respectively.

1. Verify that it is appropriate to use the normal probability distribution to approximate the binomial random variable \( X \).

2. Find the value of \( P(16) \) using the rules of the binomial random variable.

3. Using a normal distribution with \( \mu_X = 18 \) and \( \sigma_X = \sqrt{10.8} \), determine the probability that \( X \) is between 15.5 and 16.5.

4. Compare your answers to items 2 and 3.

5. Clinical trials of the popular allergy medicine Allegra-D found that 13% of the trial subjects reported headache as an adverse reaction to the drug. A follow-up study involves a random sample of 2150 allergy sufferers. What is the probability that less than 250 report headache?

   a. Find the mean and standard deviation of the binomial random variable.

   b. Determine the area on the normal distribution that approximates the variable. (Don’t forget the correction for continuity.)

   c. Find the z-score(s) for the area on the normal distribution.

   d. Determine the probability of the area.