Post Hoc Analysis – Tukey’s Test

Step 0: Verify Assumptions
Tukey’s test has five assumptions.
1. The $k$ samples are each obtained using simple random sampling.
2. The $k$ samples data independent of each other within and among the samples.
3. The $k$ populations are normally distributed.
4. The $k$ populations have equal variances.
5. A decision to reject the null hypothesis that $\mu_1 = \mu_2 = \mu_3 = \ldots = \mu_k$ was made during the one-way ANOVA.

Step 1: State the Hypothesis
A claim is made regarding pairs of population means. This claim is used to determine the following null and alternative hypotheses.

$H_0$: $\mu_i = \mu_j$

$H_1$: $\mu_i \neq \mu_j$

Step 2: Select a Level of Significance
The level of significance $\alpha$ is determined by the one selected in the one-way ANOVA.

Step 3: Calculate the Test Statistic
The test statistic for Tukey’s test is given by:

$$ q = \frac{\bar{x}_j - \bar{x}_i}{\sqrt{\frac{s^2}{2} \left( \frac{1}{n_i} + \frac{1}{n_j} \right)}} $$

where $\bar{x}_j > \bar{x}_i$,

$s^2$ is the mean square error estimate of $\sigma^2$ (MSE) from ANOVA,

$n_i$ is the sample size from population $i$, and

$n_j$ is the sample size from population $j$.

Step 4: Determine the Decision Criterion

The Classical Approach: Find the Critical Value for Tukey’s Test
The critical value for Tukey’s test using a familywise error rate $\alpha$ is given by $q_{\alpha,v,k}$

where $v = n - k$, the degrees of freedom due to error from ANOVA, and $k$ is the total number of mean being compared.

Step 5: Make a Decision
Reject the null hypothesis if $q \geq q_{\alpha,v,k}$.
Do not reject the null hypothesis if $q < q_{\alpha,v,k}$.
**Step 6: State the Conclusion**
State the conclusion of the hypothesis test based on the decision made and with respect to the pairwise claim.

<table>
<thead>
<tr>
<th>Decision</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Reject $H_0$</strong></td>
<td>There is sufficient evidence (at the $\alpha$ level) to conclude that the means of populations $i$ and $j$ are significantly different.</td>
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<tr>
<td><strong>Do Not Reject $H_0$</strong></td>
<td>There is not sufficient evidence (at the $\alpha$ level) to conclude that the means of populations $i$ and $j$ are significantly different.</td>
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