An Inquiry into the Influences of Pedagogical Content Knowledge and Epistemological stance on Teachers’ Approach to Teaching Fundamental Theorem of Calculus

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Research Question: How do a teacher’s epistemological stance and pedagogical content knowledge inform and influence the presentation and content involved in their instruction of the Fundamental Theorem of Calculus (FTC) in undergraduate courses?

Abstract
This qualitative study examined four university mathematics instructors with differing levels of educational and teaching experience. We considered data from our individual interviews with these four instructors. The analysis of these interviews led us to discover that common influences on instructors’ approach to the FTC include KCS, SCK, CCK, and personal epistemological beliefs. However, the impact of these factors may differ from instructor to instructor. The data also suggests that PCK consists of overlapping components, contrary to Hill, Ball, & Schilling (2008) construct of PCK.

Literature Review
Understanding the key ideas in mathematics is fundamental for learning mathematics. If students understand these important concepts, it opens an opportunity for the students to build upon these foundations. In Calculus, the FTC is an integral cornerstone to understanding the relationship between differentiation and integration. It gives the precise relationship between the derivative and the integral, and thus connects two main concepts of Calculus.

It is easy to lose sight of the broad and conceptual role of the FTC in an otherwise skills-oriented calculus course. Emphasis on skills is common since “skills are easy to test for, and tests for skills are easy to defend” (Schoenfeld, 2007, p. 72). However, skills-based problems often suffer from “absence of human meaning or purpose, which may inhibit both motivation and imagination” (Rutheven, 1994, p. 440), leaving “significant issues with regard to ... conceptual understanding” (Schoenfeld, 2007, p. 72). This is especially problematic since “students take tests as models of what they are to know,” so “assessment shapes what students attend to, and what they learn” (Schoenfeld, 2007, p. 72).

Methods
We contacted seven instructors, at the university in the Rocky Mountain Region, through e-mail. Each of the instructors we contacted had taught the FTC in calculus course at least once. Four of the seven instructors volunteered for our research. Individual members of our research team conducted semi-structured one-on-one interviews with the participants. These interviews were audio recorded and later transcribed. The transcribed from these interviews served as our data that we analyzed and coded for recurring themes.

Participant Biographies

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<td>Dr. Columbo possessed a PhD in pure mathematics focusing on approximation theory and complex analysis. He has experience a wide range of undergraduate and graduate courses, including: different stages in the calculus course sequence, linear algebra, abstract algebra, basic analysis, real analysis, and complex analysis.</td>
<td>Dr. Columbo's past experiences with the FTC are primarily from her teaching and educational experiences. These two aspects appeared to be the root of all Dr. Columbo's influences for her approach to teaching the FTC. Her teaching and educational experience seemed to inform her curricular content knowledge, knowledge of content and students, specialized content knowledge, and personal epistemology. Evidence that Dr. Columbo's teaching experience and educational experience are the primary influencing factors in her instructional practice aligns with current research (Kuhn, Cheney, &amp; Weinsstock, 2000; Speer, 2001).</td>
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Hector
Hector possessed a masters in mathematics with a focus on real analysis. He has been teaching courses in the Calculus sequence at the university of our research focus for just over 10 years.

Mussi
Mussi has almost four decades of teaching experience, primarily at the community college level. In particular, he has taught the calculus sequence many times. He has a masters degree in applied mathematics and is in his second year as educational mathematics doctoral studies.

Patrick
Patrick was in his fourth and final year in an educational mathematics doctoral program. He possessed a strong mathematical background focused in combinatorics and modeling. Patrick also has diverse experience in teaching undergraduate introductory level mathematics courses, such as college algebra, applied calculus, calculus, and discrete mathematics.

Hector’s experiences teaching Calculus influenced the way he approaches the FTC. His SCK and KCS were the primary factors that helped him understand students’ misconceptions and obstacles regarding the FTC, and enabled him to help students to overcome these misconceptions. He utilized his considerable SCK to generate enthusiasm and stimulate his most talented students on matters beyond the scope of the course. Hector teaching experience also informed his views on how students learn and shaped his teaching.

Mussi’s teaching of the FTC is from an epistemological point of view highly non-authoritarian. He emphasizes the differences in learning styles among students and makes accommodations for this in his teaching. A second major factor influencing his teaching of the FTC is its role in the broader curriculum and landscape of knowledge. He emphasizes how it builds on and extends ideas from Calculus I and how it relates to physics, and he omits a more theoretical treatment of the theorem and its proof with reference to later courses in Real Analysis.

Patrick’s PCK allowed him to better understand his students thinking and learning. This allowed him to consider these things in his teaching practices. We believe that his teaching is enhanced by his deep understanding of the concepts that he has demonstrated from the examples he provided from geometry, physics, and statistics. These results are consistent with the results in the existing literature.

Preliminary Conclusions

- Instructors’ educational and instructional experiences appeared to the major informing factors of their PCK and personal epistemologies. The instructors then seemed to utilize their PCK and personal epistemology to inform their approach to teaching the FTC.
- Dimensions of an instructor’s epistemology seemed to vary depending on instructional situation and mathematical topic.
- While Hill’s, Ball’s, and Schilling’s (2008) concept of PCK and subject matter knowledge consisted of disjoint, mutually exclusive subsets of teacher knowledge, our participants demonstrated informing connections among these subsets of knowledge.

Future Research

- Explore how teachers’ epistemological beliefs, PCK, and their approach to the FTC impacts students conceptual understanding of the FTC, perception of the importance of the FTC, and the students’ epistemological beliefs.
- Study how a teacher’s epistemological beliefs and PCK contributes to student achievement in undergraduate mathematics courses.
- Examine our data across faculty roles, such as graduate teaching assistant, instructor, and research professor, for differences and similarities in PCK, personal epistemology, and instructional practices.

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