

Combinations and Permutations

Math 182



Permutation

- An arrangement where order is important is called a ***permutation***.
- An arrangement where order is **not** important is called ***combination***.

Seating Arrangement

Purpose: You are a photographer sitting a group in a row for pictures. You need to determine how many different ways you can seat the group.

Part One

1. There are three people to sit down in a row. Let the colors red, blue, and green represent the three people. If **Red** is the first to sit down, show all the possible seating arrangements. (Use colored pencils to show the different arrangements.)
2. If **Blue** is the first to sit down, show all these possible arrangements.
3. If **Green** is the first to sit down, show all these possible arrangements.
4. How many total possible seating arrangements are there for three people?

Part Two

5. There are five people in the group. Let the colors red, blue, green, yellow, and purple represent the five people. How many people could sit down first for the picture?

6. If Red sits down, how many people are left to sit down?

7. If Blue sits down, how many people are left to sit down?

8. If Green sits down, how many people are left to sit down?

9. If Yellow sits down, how many people are left to sit down?

10. Multiply how many can sit down at each turn. How many possible seating arrangements are there for five people?

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9. If Yellow sits down, how many people are left to sit down?
10. Multiply how many can sit down at each turn. How many possible seating arrangements are there for five people?

$$P = 5 \times 4 \times 3 \times 2 \times 1 = 120 \text{ ways}$$

- A seating arrangement is an example of a **permutation** because the arrangement of the “n” objects is in a specific order. The order is important for a permutation.
- When the order does not matter, it is a **combination**, because you are only interested in the group.

Extend:

- . Twelve people need to be photographed, but there are only five chairs. (The rest of the people will be standing behind and their order does not matter.) How many ways can you sit the twelve people on the five chairs?

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$$12 \times 11 \times 10 \times 9 \times 8 = 95040 \text{ ways}$$

Permutation

- An arrangement where order is important is called a ***permutation***.
- Example: Mario, Sandy, Fred, and Shanna are running for the offices of president, secretary and treasurer. In how many ways can these offices be filled?

$$4 \times 3 \times 2 = 24.$$

The offices can be filled 24 ways.

Combination

- An arrangement where order is **not** important is called ***combination***.
- Example: Charles has four coins in his pocket and pulls out three at one time. How many different amounts can he get?

$${}_4C_3 = \frac{{}_4P_3}{3!} = \frac{4 \times 3 \times 2}{3 \times 2 \times 1} = \frac{24}{6} = 4$$

- Determine if the situation represents a permutation or a combination:
 1. In how many ways can five books be arranged on a book-shelf in the library?
 2. In how many ways can three student-council members be elected from five candidates?
 3. Seven students line up to sharpen their pencils.
 4. A DJ will play three CD choices from the 5 requests.

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combination
 3. Seven students line up to sharpen their pencils.
permutation
 4. A DJ will play three CD choices from the 5 requests. **combination**

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$${}_5C_3 = \frac{5 \times 4 \times 3}{3!} = \frac{60}{6} = 10 \text{ ways}$$

- Find the number of events:

3. Seven students line up to sharpen their pencils.

$$P=7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040 \text{ ways}$$

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Fundamental Counting Principle.

If Act 1 can be performed in m ways,
and Act 2 can be performed in n ways
no matter how Act 1 turns out,
then the sequence Act 1 and Act 2
can be performed in $m \cdot n$ ways.

- ***Example 1:*** Eight horses-Alabaster, Beauty, Candy, Doughty, Excellente, Friday, Great One, and High 'n Mighty- run a race.
- In how many ways can the first three finishers turn out?

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$$\underline{8} \times \underline{7} \times \underline{6} = 336 \text{ ways}$$

- Solution:

Extending the Fundamental Counting Principle to three acts, finishing first ("Act 1 ") can happen in 8 ways, there are then 7 ways in which finishing second ("Act 2") can occur, and finally there are 6 ways in which third place ("Act 3 ") can be filled, so the first three finishers could occur in $8 \cdot 7 \cdot 6 = 336$ ways.

- ***Example 2:*** How many ways can 10 tosses of a coin turn out?

- *Solution:*

Each of the 10 acts can occur in two ways (H or T).

So there are $2^{10} = 1024$ different sequences possible.

- ***Example 3:*** Given a list of 5 blanks, in how many different ways can A, B, and C be placed into *three* of the blanks, one letter to a blank? (Two blanks will be empty.)

- *Solution: There are 5 choices of a blank for A, then 4 for B, and finally 3 for C. So there are $5 \cdot 4 \cdot 3 = 60$ ways in which the three letters can be placed in the five blanks.*

- Order matters in spelling and numbers- RAT and TAR are different orders of the letters A, R, and T, and certainly have different meanings, as do 1234 and 4231. These are permutations.
- But many times order is not important, these are **combinations**. ABC, ACB, BAC, BCA, CAB, and CBA are six different permutations of the letters A, B, and C from the alphabet, but they represent just one combination.

- Choose one combination of four different letters from the alphabet.
- How many permutations does this one combination give?

- ***Example 4:*** In how many different ways could a committee of 5 people be chosen from a class of 30 students?

- 5 positions to be filled

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- 30 people to chosen from
- 29 left to choose from, etc
- Therefore 30x29x28x27x26 = 17,100,720 permutations
- Divide out repeats of 5!
- So 142,506 combinations

- ***Example 5:*** If the first chosen would be chair, the next one vice-chair, then secretary, and finally treasurer, in how many different ways could a committee of four be chosen?

- ***Example 7:*** In a toss of 10 different honest coins, what is the probability of getting exactly 5 heads?

- ***Example 8:*** In 10 tosses of a "loaded" coin, with probability of heads = 0.7, what is the probability of getting exactly 5 heads?