Group Assignment #4

RATES OF CHANGE

Lee is training for a marathon. Every morning he leaves his house to go for a 10-mile out-and-back run, which consists of running 5 miles from home and returning on the same path. Lee begins warming up on the run over the first 2 miles, which takes 16 minutes. In the next 21 minutes, Lee runs 3 miles, after which point he turns around and begins his run home. Lee begins to feel good and he completes the next 4 miles in 26 minutes. On the last mile, he cools down, takes his time, and finishes the last mile in 10 minutes.

1) Using the data provided in the story above, create a table that describes the time (minutes) Lee runs in relation to his distance (miles) from home. Be sure to include all the data provided above in your table. Be careful deciding which of the measurements is the independent variable and which is the dependent variable.

<table>
<thead>
<tr>
<th>time (min)</th>
<th>mile (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>16</td>
<td>2</td>
</tr>
<tr>
<td>37</td>
<td>5</td>
</tr>
<tr>
<td>63</td>
<td>1</td>
</tr>
<tr>
<td>73</td>
<td>0</td>
</tr>
</tbody>
</table>

2) Create a graph that appropriately represents the data in your table. Consider the following question: Should your graph be continuous or discontinuous?

The graph is continuous.

3) Is the graph you created one-to-one? Explain how you know.

The graph fails to be one-to-one since there are two x-values for one y-value. (Fails horizontal line test)
4) Using either your table or your graph, determine the slope (rate of change) of each of Lee’s different paced intervals. Hint: There are 4 different intervals.

\[ I_1 (0, 16) : \frac{2-0}{16-0} = \frac{2}{16} = \frac{1}{8} \]
\[ I_2 (16, 37) : \frac{5-2}{37-16} = \frac{3}{21} = \frac{1}{7} \]
\[ I_3 (37, 63) : \frac{9-5}{37-63} = -\frac{4}{26} = -\frac{2}{13} \]
\[ I_4 (63, 73) : \frac{10-9}{73-63} = \frac{1}{10} \]

5) Determine the formula representation of the function for each interval of your graph. How might you use these formulas to create a single function formula that describes Lee’s run?

\[ I_1 : m = \frac{1}{8}, \quad y - 0 = \frac{1}{8} (x - 0) \Rightarrow y = \frac{1}{8} x \]
\[ I_2 : m = \frac{1}{7}, \quad y - 2 = \frac{1}{7} (x - 16) \Rightarrow y = \frac{1}{7} x - \frac{16}{7} + 2 \Rightarrow y = \frac{1}{7} x - \frac{2}{7} \]
\[ I_3 : m = -\frac{2}{13}, \quad y - 5 = -\frac{2}{13} (x - 37) \Rightarrow y = -\frac{2}{13} x + \frac{74}{13} + 5 \Rightarrow y = -\frac{2}{13} x + \frac{139}{13} \]
\[ I_4 : m = -\frac{1}{10}, \quad y - 0 = -\frac{1}{10} (x - 73) \Rightarrow y = -\frac{1}{10} x + \frac{73}{10} \]

\[ f(x) = \begin{cases} 
\frac{1}{8} x, & [0, 16) \\
\frac{1}{7} x - \frac{2}{7}, & [16, 37) \\
-\frac{2}{13} x + \frac{139}{13}, & [37, 63) \\
-\frac{1}{10} x + \frac{73}{10}, & [63, 73] 
\end{cases} \]

6) Using the information in problem #4, calculate the average rate of change (average slope). Start by multiplying the rate of change for each interval by the time elapsed in that interval. Add these values together and then divide by the total elapsed time for Lee’s run.

\[ \frac{\frac{1}{8} (16) + \frac{1}{7} (21) - \frac{2}{13} (26) - \frac{1}{10} (10)}{73} = \frac{2 + 3 - 4 - 1}{73} = \frac{0}{73} = 0 \]

7) What explanation do you have for the average rate of change you found in problem #6? How does this make sense in terms of the story?

The y-value of the starting point 0 miles from home is the same as the ending point 0 miles from home, so the change in y-values from that starting time to the ending time of the run is 0.
8) Is there an easier or more efficient method for calculating the average rate of change for the data above? If so, what is it? Hint: There is.

\[
\begin{align*}
(0, 0) & \quad \text{start point} \\
(73, 0) & \quad \text{end point}
\end{align*}
\]

average rate of change \[= \frac{0 - 0}{0 - 73} = \frac{0}{-73} = \frac{0}{73}\]

9) Using the data provided in the story above, create a new table that describes the time (minutes) Lee runs in relation to the distance (miles) he ran. Be sure to include all the data provided above in your table. Why is this table different from your table in problem #1?

\[
\begin{array}{c|c}
\text{time (t)} & \text{miles (m)} \\
\hline
0 & 0 \\
16 & 2 \\
37 & 5 \\
63 & 9 \\
73 & 10 \\
\end{array}
\]

10) Create a graph that appropriately represents the data in your table.

11) Is the graph you created one-to-one? Explain how you know.

yes, each \( y \)-value has one unique corresponding \( x \)-value. Passes horizontal line test

12) Determine the formula representation of the function for each interval of your graph. Use these formulas to create a single function formula that describes Lee’s run.

\[
\begin{align*}
I_1: m &= \frac{1}{8}, \quad y = \frac{1}{8}x \\
I_2: m &= \frac{1}{7}, \quad y = \frac{1}{7}x - \frac{2}{7} \\
I_3: m &= \frac{5}{13}, \quad y - 5 = \frac{5}{13}(x - 37) \Rightarrow y = \frac{5}{13}x - \frac{79}{13} + 5 \Rightarrow y = \frac{5}{13}x - \frac{4}{13} \\
I_4: m &= \frac{1}{10}, \quad y - 0 = \frac{1}{10}(x - 73) \Rightarrow y = \frac{1}{10}x + \frac{73}{10} \\
\end{align*}
\]

\[
\begin{align*}
f(x) &= \begin{cases} 
\frac{1}{8}x, & [0, 16) \\
\frac{1}{7}x - \frac{2}{7}, & [16, 37) \\
\frac{5}{13}x - \frac{4}{13}, & [37, 63) \\
\frac{1}{10}x + \frac{73}{10}, & [63, 73] 
\end{cases}
\end{align*}
\]
13) Using your preferred method for calculating average rate of change, determine the average rate of change for the new function.

\[
\frac{10 - 0}{73 - 0} = \frac{10}{73}
\]

\((0, 0)\) - starting point

\((73, 10)\) - ending point

14) What explanation do you have for the average rate of change you found in problem #13? How does this make sense in terms of the story?

This describes the average rate of change in total miles run over total time run.

15) What accounts for the difference between the two different average rates of change?

The difference in average rates of change come from the change in calculating mileage. In the first story distance was measured in miles from home, and in the second story distance was measured in total miles ran.

Suggested Homework Problems
Sec. 1.3: 2, 6, 7, 10, 12, 16, 18, 25, 26, 27

NOTE: You can find the formula for average rate of change in your textbook on page 14.