Group Assignment #20

APPLICATION OF DERIVATIVES:
FINDING LOCAL MAXIMA AND MINIMA
AND INFLECTION POINTS

1) Recall our previous work with derivatives and their graphs. How could we
determine at which $x$-value(s) a function $f(x)$ has either a local maxima or a local
minima? Find where the graph of the derivative function
crosses the $x$-axis.

2) How can we use the derivative function $f'(x)$ to determine at which $x$-value(s) a
function $f(x)$ has either a local maxima or a local minima?
   Find the derivative formula of $f(x)$, and
   then set it equal to zero and solve
   for $x$.

3) Using only the derivative function, how can we tell which $x$-value is a local
maxima? Local minima? Looking at the graph of the
derivative function, if the graph goes from positive
to negative $\Rightarrow$ local max. If graph goes from negative
to positive $\Rightarrow$ local min.

4) How might we find the exact point(s) $(x, f(x))$ at which the function $f$ has a local
maxima? Local minima?
   Find what $x$-value makes $f'(x) = 0$, then
determine if the $x$-value is a local max or
min. Then to find the coordinate point,
plug the $x$-value into the original function,
Definition:
For any function \( f \), a point \( p \) in the domain of \( f \) where \( f'(p) = 0 \) or \( f'(p) \) is undefined is called a critical point of the function. In addition, the point \( (p, f(p)) \) on the graph of \( f \) is also called a critical point. A critical value of \( f \) is the value, \( f(p) \), of the function at a critical point, \( p \).

First Derivative Test for Local Maxima and Minima
Suppose \( p \) is a critical point of a continuous function \( f \).
- If \( f \) changes from decreasing to increasing at \( p \), then \( f \) has a local minimum at \( p \).
- If \( f \) changes from increasing to decreasing at \( p \), then \( f \) has a local maximum at \( p \).

Second Derivative Test for Local Maxima and Minima
Suppose \( p \) is a critical point of a continuous function \( f \), and \( f'(p) = 0 \).
- If \( f \) is concave up at \( p \), then \( f \) has a local minimum at \( p \).
- If \( f \) is concave down at \( p \), then \( f \) has a local maximum at \( p \).

Definition:
A point at which the graph of a function \( f \) changes concavity is called an inflection point of \( f \).

1) Recall our previous work with derivatives. How could we determine the concavity of the original function \( f \)?

We can look at the graph of the second derivative, where the graph is positive we know that the original function is concave up and where the graph is negative the original function is concave down.

2) How could we determine when the original function \( f \) changed concavity?

The concavity changes where the graph of the second derivative crosses the \( x \)-axis.

3) Now using these ideas, if you were given some formula representation of a function \( f(x) \) how might you be able to determine the \( x \)-value(s) at which the function changes concavity?

Find the second derivative function \( f''(x) \), and set \( f''(x) = 0 \), and then solve for \( x \).

4) How might you be able to determine the exact point \( (x, f(x)) \) at which the function \( f \) changes concavity?

After finding the \( x \)-value where you have an inflection point, plug this \( x \)-value back into the original function to find its corresponding \( y \)-value.
Now using your previous work answer the following:

Suppose you have a function \( g(t) \), and that we know \( g(t) \) has a local maximum at \( t = -2 \) and local minimums at \( t = -7 \) and \( t = 1 \). Also, we know \( g(-7) = 0 \), \( g(-2) = 4 \), and \( g(1) = -3 \).

1) What does this information tell us about our original function \( g(t) \)?

   on the original function \( g(t) \) we have a local max at \((-2, 4)\) and local min at \((-7, 0)\) and \((1, -3)\).

2) What does this information tell us about the derivative function \( g'(t) \)?

   - at \( t = -2 \) the derivative function crosses the x-axis going from positive to negative. \( (g'(-2) = 0) \)
   - at \( t = -7 \) and \( t = 1 \) the derivative function crosses the x-axis going from negative to positive. \( (g'(-7) = g'(1) = 0) \)

After further work we discover that \( g(t) \) has inflection points at \( t = -4 \) and at \( t = 0 \). We also learn that \( g(-4) = 2 \) and \( g(0) = -1 \).

1) What does this information tell us about the original function \( g(t) \)?

   on the original function \( g(t) \), we have inflection points at \((-4, 2)\) and \((-1, 1)\).

2) What does this information tell us about the second derivative function \( g''(t) \)?

   At \( t = -4 \) and \( t = 0 \) the second derivative crosses the x-axis. \( (g''(-4) = g''(0) = 0) \)
Now using the above information create a sketch of a graphical representation of the function \( g(t) \).

For the following functions complete the following tasks:
(a) The critical points of the function.
(b) The critical value of the function.
(c) The inflection points of the function.
(d) Create a sketch of a graphical representation of the function.

1) \( f(x) = x^3 - 12x \)
   \[ f'(x) = 3x^2 - 12 \]
   \[ 3x^2 - 12 = 0 \]
   \[ x = \pm \sqrt{4} = \pm 2 \]

2) \( g(x) = x^2 + x - 6 \)
   \[ g'(x) = 2x + 1 \]
   \[ 2x + 1 = 0 \]
   \[ x = -\frac{1}{2} \]

\[ g(-\frac{1}{2}) = -6.25 \leftarrow \text{critical value} \]

\[ g''(x) = 2 \Rightarrow \text{always concave up} \Rightarrow \text{no inflection point} \]
3) \( h(x) = e^x - 10x \)

\[ h'(x) = e^x - 10 \]
\[ e^x - 10 = 0 \]
\[ e^x = 10 \]
\[ \ln(e^x) = \ln(10) \]
\[ x = \ln(10) \]

Critical point

\[ h'(\ln(10)) = e^{\ln(10)} - 10(\ln(10)) \leq \text{critical value} \]
\[ = 10 - 10(\ln(10)) \]

\[ h''(x) = e^x \]

\( e^x \) is always positive for every \( x \) \( \Rightarrow h''(x) = e^x \Rightarrow \) concave up always

\( \Rightarrow \) no inflection pt.

**Suggested Homework Problems:**

Section 4.2: #1-7, 10, 11-19 odd, 22, 23, 32

Section 4.3: # 1-5, 11, 15, 17, 23, 33, 39