Exam 3

No notes, books, or other materials allowed. Calculators are acceptable (no sharing).

Please turn off cell phones and put away all other electronic devices besides your calculator.

Be sure to read all the directions carefully.

SHOW ALL WORK

For grading purposes:

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Name: [Student Name]

[Signature]
1. Kim continues her research involving oceanic waves. She is now focused on the heights of tsunami waves that have occurred during recorded history. During her studies, Kim stumbled upon the largest tsunami wave ever documented. On July 9, 1958 an earthquake measuring 8.3 on the Richter scale caused a landslide in Lituya Bay, Alaska. The large amounts of earth hitting the water resulted in an immense wave. Scientific buoys in the area recorded the height of the wave in feet over a period of time in minutes since the landslide as the following function:

\[ h(t) = -10t^2 + 240t + 280 \]

Help Kim determine how long after the landslide hit the water that the wave reached its crest (i.e. its highest point) and what the height of the wave was at this time. (10 pts)

\[ h'(t) = -20t + 240 \]
\[ -20t + 240 = 0 \]
\[ t = 12 \text{ minutes} \]

\[ h''(t) = -20 \]
since \( h''(t) = -20 \) for all \( t \) we know it is concave down everywhere
\[ \Rightarrow t = 12 \text{ is local max} \]

2. A few of your fellow classmates are practicing on their derivative skills. They have encountered a problem that they are struggling with. The function they are working with is given below:

\[ f(x) = 4(x^5 - \ln(2x))^3 \]

Your classmates believe the derivative of \( f(x) \) to be the following:

\[ f'(x) = 3(x^5 - \ln(2x))^2 \]

Your classmates ask you if their derivative is correct. If they are correct, explain to them how you know their derivative is correct. If they are not correct, explain why they are not correct, then provide the correct derivative and explain how you arrived at your answer. (10 pts)

Their derivative is incorrect. We can tell they are incorrect since to find the derivative of \( f(x) = 4(x^5 - \ln(2x))^3 \) we would use the chain rule, which involves taking the derivative of the function inside the composition (inside the parentheses), and the derivative of \( x^5 - \ln(2x) \) is nowhere to be found in their derivative. Also, they forgot to keep the constant coefficient (9) with their derivative. Correct derivative is the following (use chain rule):

\[ f'(x) = 4 \cdot 3 \cdot (x^5 - \ln(2x))^2 \cdot (5x^4 - \frac{2}{2x}) \]
\[ = 12 \cdot (x^5 - \ln(2x))^2 \cdot (5x^4 - \frac{1}{x}) \]
3. A new running company, Tarahumara Feet, entered the shoe market recently. The company specializes in minimalist-style running shoes for runners interested in running barefoot, but do not have feet durable enough to run on pavement and other rocky surfaces. Starting up the new company required substantial capital, but the shoes are fairly inexpensive to produce. An accountent determined the company operated with a cost function $C(q) = 225,000 + 0.15q^2$. Tarahumara Feet studied their competition and noticed that running shoes are not sold cheaply, this allows them to operate at a revenue function $R(q) = 90q$.

(a) Determine the company’s marginal cost when they produce 285 shoes. 

$$C'(285) = 85.5$$

Thus $$C'(285) = 85.5$$

(b) Determine the company’s marginal revenue when they sell 285 shoes. 

$$R'(285) = 90$$

Therefore $$R'(285) = 90$$

(c) Based on the marginal cost and marginal revenue of the company, should the company produce and sell 286 shoes? Explain your answer. 

(5 pts)

Recall marginal profit equals marginal revenue minus marginal cost. (i.e. $MP = MR - MC$) so for $q = 285$

$$MP = R'(285) - C'(285) = 90 - 85.5 = 4.5$$. Since $MP = 4.5 > 0$ at $q = 285$, we know that if we produce and sell 286 shoes our profits increase $9.50.

The company should produce and sell 286 shoes.

(d) Determine the quantity of shoes produced and sold that would maximize profits. What is the total profit at this production? 

(10 pts)

Since $P''(q) = -0.3$ 

$$P'(q) = 90 - 0.3q$$

Therefore, we know at the critical point $q = 300$, we have a local max (max profit). Then to find the company’s profits for producing and selling 300 shoes:

$$P(300) = 90(300) - 225,000 - 0.15(300)^2 = -21,150$$

4. For which $x$ values is the graph $f(x) = x^4 - 4x^3$ flat, i.e. has slope 0? Use the second derivative to determine whether the graph of $f(x)$ is concave up or down at $x = 1$. (10 pts)

$$f'(x) = 4x^3 - 12x^2$$

$$= 4x^2(x - 3)$$

$$4x^2(x - 3) = 0$$

$$\Rightarrow x = 0, x = 3$$

Since derivative is zero at $x = 0$ and $x = 3$, we know the slope of $f(x)$ is zero at these points.

$$f''(x) = 12x^2 - 24x$$

$$f''(1) = 12(1) - 24(1) = -12$$

Since the second derivative is negative at $x = 1$, we know $f(x)$ is concave down at $x = 1$. 

5. For the function \( f(x) = 2x^3 + x^2 - 4x - 2 \) complete the following tasks:

(a) Find \( f'(x) \) and \( f''(x) \). (5 pts)

\[
f'(x) = 6x^2 + 2x - 4
\]
\[
f''(x) = 12x + 2
\]

(b) Find the critical point(s) for \( f(x) \). (5 pts)

\( 6x^2 + 2x - 4 = 0 \)
\[
(6x - 4)(x + 1) = 0
\]
\[
x = \frac{2}{3}, \quad x = -1
\]

(c) Find the coordinates for the local maximum and minimum of \( f(x) \), if they exist. Be sure to label which coordinate(s) are local maximums and which coordinate(s) are local minimums. (5 pts)

\[
f''(\frac{2}{3}) = 12(\frac{2}{3}) + 2 = 10 \quad \Rightarrow \text{concave up} \quad \Rightarrow \text{local min} \quad \Rightarrow \quad x = \frac{2}{3}
\]
\[
f''(-1) = 12(-1) + 2 = -10 \quad \Rightarrow \text{concave down} \quad \Rightarrow \text{local max} \quad \Rightarrow \quad x = -1
\]

\[
f\left(\frac{2}{3}\right) = -\frac{98}{27}
\]
\[
\left(\frac{2}{3}, -\frac{98}{27}\right) \text{ local min}
\]
\[
f(-1) = 1
\]
\[
(-1, 1) \text{ local max}
\]

(d) Find the coordinates of the inflection point(s) for \( f(x) \). (5 pts)

\[
f''(x) = 12x + 2
\]
\[
12x + 2 = 0
\]
\[
x = -\frac{1}{6}
\]
\[
f\left(-\frac{1}{6}\right) = -\frac{71}{54}
\]

(e) Find the coordinates for the global maximum and minimum of \( f(x) \) within the domain \([-2,2]\). (5 pts)

\[
f(2) = -6
\]
\[
(-2, -6) \text{ global minimum}
\]
\[
f(2) = 10
\]
\[
(2, 10) \text{ global maximum}
\]
\[
f(-1) = 1
\]
\[
f\left(\frac{2}{3}\right) = -\frac{98}{27}
\]
\[
\left(\frac{2}{3}, -\frac{98}{27}\right) \text{ global minimum}
\]

(f) Sketch the graph of the function \( f(x) \) for \( x \in [-2,2] \). (5 pts)
6. Compute the derivatives of the following functions.

(a) \( f(x) = (x - 2)(2x + 3) \)  
\[
f'(x) = 1(2x+3) + 2(x-2)
\]

(b) \( g(x) = \frac{2x^2}{1-x^3} \)  
\[
g'(x) = \frac{4x(1-x^2) - 2x^2(-3x^2)}{(1-x^3)^2}
\]

(c) \( s(x) = 3e^{2x^2-6x+1} \)  
\[
s'(x) = 3(4x-6)e^{2x^2-6x+1}
\]

BONUS: (5 pts)

Create a graphical representation of the wave described in question #1 using the function given. Be sure to use an appropriate domain for your graph.