

For up to 10 points added to your exam 3 score, you may submit worked-out solutions to the following problems by **Wednesday, July 5th at 10am**. Attach your work separately.

People who navigate the globe (pilots, movie stars, etc) sometimes like to find distances. The process starts with drawing a *great circle* on the earth which connects the two points. The radius of the great circle will be roughly equal to the average radius of the earth: $r = 3959.87$ miles. The drawing below shows an example of a great circle connecting two locations on earth along with the angle formed between them.



To calculate earth distances, you first need to know latitude and longitude coordinates. Say $A = (\lambda_1, \delta_1)$ and $B = (\lambda_2, \delta_2)$ are the (latitude, longitude) coordinates for two locations. We can indirectly find the angle θ between A and B using a formula from spherical geometry:

$$\cos(\theta) = \sin(\lambda_1) \sin(\lambda_2) + \cos(\lambda_1) \cos(\lambda_2) \cos(\delta_1 - \delta_2)$$

Important Notes:

- The formula considers *longitudes* that are *west* of the prime meridian and *latitudes* that are *south* of the equator to be *negative*.
 - The formula does not apply if the two locations have the same longitude. In that case, the angle between the points is just $\lambda_1 - \lambda_2$.
 - Precision is important in using this formula. I would keep at least *6 digits of precision* in my calculations.
1. Start by choosing any 3 of the 20 largest cities in the world. Let's call these cities A, B, and C. (Two good lists are www.infoplease.com/ipa/A0762524.html and www.infoplease.com/ipa/A0884418.html)
 - (a) Find the (latitude, longitude) coordinates for each of your chosen cities with internet searches. Be careful to write the coordinates in decimal form and use minus signs when appropriate.
 - (b) Using the formula, compute $\cos(\theta)$ for the great circle connecting cities A and B. Then, use the inverse cosine to find the angle θ . Finally, apply the arc length formula ($s = r\theta$) to compute the distance between the two cities. (Hint: You may need to convert θ to radians)
 - (c) Repeat the exercise to find the distances between cities B and C, and then A and C. How accurate do you think your results are?
 - (d) How do you think the distances you computed would change if you thought the earth was flat? When might it be reasonable to assume a flat earth?

- Using a website (like <http://geocoder.us>), look up the latitude and longitude coordinates of (A) your current address and (B) a favorite location of yours in North Dakota (funny locations are good). Use the formula to find θ and hence the distance between the locations. Does the distance you computed seem reasonable? Why or Why not?

Another (equivalent) formula to find θ is called the haversine formula:

$$\sin^2\left(\frac{\theta}{2}\right) = \sin^2\left(\frac{\lambda_1 - \lambda_2}{2}\right) + \cos(\lambda_1)\cos(\lambda_2)\sin^2\left(\frac{\delta_1 - \delta_2}{2}\right)$$

- Recompute the distance in the previous problem using the haversine formula. Keep in mind that you'll need to first compute the right hand, then take a square root, then use inverse sine, then double the angle. Which result to you think is more accurate? Why?

- **Challenge**** Prove that the two formulas for finding θ are equivalent.
[Hint: I would start with the haversine formula and use the half angle formulas and an angle difference identity to get the other formula.]