MED 610 Fall 2006 Mid-Term Essays Joe Champion

# The Message about Mathematics Education in *the Principles and Standards for School Mathematics*

The largest organization supporting mathematics education in the world is the National Council of Teachers of Mathematics ([NCTM], 2006). With over 100,000 members in the U.S. and Canada, the documents produced by the NCTM have contributed to major and sometimes controversial changes in mathematics education over the past decade. A catalyst for much of the changes is the influential book *Principles and Standards for School Mathematics* (NCTM, 2000), whose principles have been incorporated into nearly every level of preK-12 mathematics curriculum and instruction. As the title indicates, the *Principles and Standards* is a document that details six principles, or “basic precepts that are fundamental to high-quality mathematics education” (NCTM, 2000, p.6), as well as ten standards that describe elements of curriculum that mathematics students should be enabled to learn in school. In the summary that follows, I will describe the vision of mathematics education in the *Principles and Standards* and how it arises from scientifically supported educational values (principles) and goals (standards). I will conclude by briefly summarizing some of the criticisms of the many changes that the *Principles and Standards* have contributed to producing in mathematics education.

The vision of the NCTM in *Principles and Standards* is one of an engaging classroom where flexible students work in a variety of ways to accomplish meaningful and challenging mathematical learning goals. An underlying assumption of the document is that students seek meaning in their world and will benefit from the tools, perspectives, and skills that come with quality mathematics instruction. Students who participate in the kind of mathematical learning recommended by the *Principles and Standards* will be better prepared to participate in culture, to contribute to an increasingly technologically sophisticated work force, and to value the intellectual heritage of mathematics and science. The intended audience of the book is primarily mathematics teachers, but includes mathematics curriculum developers and educational policy makers.

According to the NCTM, six principle core values are associated with the successful teaching and learning of mathematics. They include the Equity Principle, the Curriculum Principle, the Teaching Principle, the Learning Principle, the Assessment Principle and the Technology Principle. Equity refers to the expectation that mathematics teachers should maintain high expectations for all students in their class while providing any necessary support to those who need it. The Curriculum Principle centers on the idea that the mathematical activities engaged in by students should be focused, coherent, and integrated across the grade levels. In order to effectively challenge and support students’ learning, the Teaching Principle highlights the necessary work a teacher must do to understand what students know and need to learn. The idea is that a teacher is most effective when they can accurately gauge their students’ understanding and can thus implement strategies that will challenge the students to grow in their understanding and proficiency.

The view of cognition put forth in the Learning Principle aligns with perspectives in constructivism and social cognition, and stresses the mechanisms of building new knowledge from prior knowledge and learning for understanding. It is in the Learning Principle where some traditional methods of mathematics instruction that focus on repetition and memorization of procedures may fall short, since “the alliance of factual knowledge, procedural proficiency, and conceptual understanding” (NCTM, 2000, p. 19) is critical to meaningful learning in mathematics. The Assessment Principle centers around the many ways that formative and summative evaluations of progress in mathematics can and should provide useful information to both teachers and students. Assessment is one area of the *Principles and Standards* that was very rewarding for me to read. I usually dislike giving and grading traditional college mathematics exams and was pleasantly surprised to learn that educational research supports alternative forms of assessment as ways to provide meaningful feedback and promote learning. Finally, the Technology Principle specifically points to the transformative role that technology has recently played in changing both the content of school mathematics and the way that it is studied.

The largest section of *Principles and Standards* focuses on recommended curriculum that will help all students to build a solid foundation of mathematics understanding and proficiency. General goals associated with five content standards and five process standards are outlined in Chapter 3 of the text, followed by grade-specific recommendations in Chapters 4-7. Although the ten standards apply across all grades, each strand should not have equal emphasis in every grade band (Van de Walle, 2006). For instance, the first of the five content standards, Number and Operations, is typically emphasized most heavily in grades preK-5 because of the developmental needs of the youngest students as they learn basic arithmetic and the properties of numbers. The remaining four content standards are Algebra, Geometry, Measurement, and Data Analysis and Probability. *Principles and Standards* includes a thorough summary of the general and grade specific goals of each of instruction for each of the five content standards, but it is important to reemphasize the recommendation of the NCTM that *all* the content strands be incorporated at *every* level of mathematical sophistication in the schools. For example, the Algebra standard states that a student should first encounter qualitative and quantitative changes in the preK-2 grade band, then gradually develop sophisticated methods for representing changes in variables as they progress through the mathematical program.

While the Number and Operations, Algebra, Geometry, Measurement, and Data Analysis and Probability standards provide detailed suggestions for the content of mathematics instruction at all grade levels, the five process standards focus on the ways that learners acquire and use mathematical knowledge. Research based approaches for supporting Problem Solving, Reasoning and Proof, Communication, Connections, and Representation of mathematics content provide mathematics teachers with a wealth of strategies for instruction. Problem Solving refers to the process of “engaging in a task for which the solution method is unknown” (NCTM, 2000, p.51). Well-chosen problem solving tasks can help students build new knowledge while learning to apply and modify strategies and reflect on their understanding of mathematics. Reasoning and Proof activities help students develop the invaluable logical structures in mathematics that relate to making conjectures and establishing mathematical truth through justification and deduction. The importance of using mathematical language to express your ideas and support organization of concepts is also stressed in the Communication standard. Finally, the Connections standard and the Representation standard are listed separately in the text to highlight the dual importance placed on (1) recognizing the many connections within mathematics as well as between mathematics and other disciplines, and (2) selecting, applying, and translating among representations and models of mathematical ideas.

Now that I have summarized the vision of mathematics teaching and learning in *Principles and Standards* as well as some of the details of the six principles and ten standards, I think it is now important to detail some of the criticisms aimed at the changes it has inspired in mathematics instruction. While research in mathematics education research largely supports the recommendations of the NCTM in *Principles and Standards*, the publication is often linked with the passage and implementation of the No Child Left Behind Act, which presses for increased standardized testing, higher student achievement in mathematics, and increased teacher accountability (Van de Walle, 2006). Many of the federally mandated reforms in state and local mathematics instruction have been tied to the recommendations of the NCTM in *Principles and Standards*. Thus, any frustrations with standardized testing implementation or pressures to hold districts accountable for student progress on mathematics exams are likely to be associated with the NCTM and their publication. According to Kohn (2000), the primary opposition to the “tougher standards” movement has come from the teachers themselves, with an increasing number of parents who are dissatisfied with their “children’s schools [being] transformed into giant test prep centers” (Kohn, 2000, p.6). Kohn and others point to an ironic outcome of the efforts mandated by No Child Left Behind—while the NCTM strongly opposes mathematics instruction resulting in the memorization of procedures and facts without understanding, high-stakes testing may actually reward teachers and school districts that “teach to the test.”

Apart from the link to recent reforms of the education system in America, some mathematics teachers have expressed dissatisfaction with the vision of teaching and learning in the *Principles and Standards*. For example, a number of teachers claim that requiring a teacher to address all of the standardized curriculum of a course can result in unintended negative consequences for learning. If an instructor must cover an unreasonable amount of content, they may feel forced to use ineffective instruction techniques. Some teachers also point to what they view as implicit assumption in *Principles and Standards* that students are motivated to learn mathematics, which is unfortunately not always true in the classroom.

This concludes my summary of the message about mathematics education presented by the NCTM in the *Principles and Standards*, along with some of the common criticisms of the implementation of the recommendations. I hope you have learned something… I sure did.

## Constructivism as a Knowledge Claim

Knowledge claims refer to the assumptions that a researcher makes about how and what they will learn during an investigation (Creswell, 2003). While assumptions about knowledge in research can take many forms and probably lie along a kind of continuum, Creswell identifies four major paradigms that inform knowledge claims in research. There are clearly defined differences between each of the assumptions made about knowledge between postpositivism, advocacy/participatory, constructivism, and pragmatism. Postpositivism, for example, is rooted in the philosophy of determinism and is generally manifested in controlled experiments that test hypotheses about variables. Like postpositivism, constructivism is an overarching paradigm for collecting and interpreting data in research, but it carries very different assumptions about what constitutes knowledge. Constructivism includes an interpretivist view of epistemology (the study of knowledge and truth). The interpretivist philosophy of knowledge rejects the objectivist (e.g. postpositivist) view that all knowledge is independent of the individual, and is thus observable; interpretivists claim instead that what we identify as truth is dependent on our frame of reference (Driscoll, 2000).

Partly due to their interprevist view of epistemology, constructivists believe that all individuals seeking meanings in their surroundings construct potentially valid and diverse personal truths from their experiences. This view that there may be multiple valid representations of an event, and that the meaning each individual attaches to the event is significant, leads researchers in the constructivism camp to look for the complexity of meanings formed by participants. Unburdened by assumptions that require them to find the “true interpretation” of their data, constructivist researchers instead look to identify the processes and experiences that affect their participants' perceptions of events. So, instead of testing *a priori* theories of a phenomenon, constructivism holds that inductive reasoning requires an observer to collect and analyze data on participants' constructed meanings until themes emerge from the data. As the researcher interprets themes of the socially constructed meaning, a theory of the events emerges and ultimately serves as the knowledge gained from the study.

## Situated Cognition as a Learning Theory

Situated cognition, along with constructivism, information processing, and behaviorism, is one of the dominant theories of learning incorporated into mathematics education research. Of the four dominant learning theories, social cognition is the newest and shares many of the assumptions associated with constructivism. Like constructivism and information processing, situated cognition acknowledges the idea that learning must take into account the internal mental processes of the learner. However, an important contrast between situated cognition and other theories of learning is the prominent place that the environment of learning plays in situated cognition (Driscoll, 2000). While situated cognition proponents acknowledge the fundamental role of internal processes in the mind, they also point to the importance of the physical and social contexts surrounding a community of learning and claim that the *activity* plays a central role in the learning of the person engaging in it (Putnam, & Borko, 2000). In other words, the setting of learning—not just the person doing the learning—is an essential variable related to learning. According to Driscoll (2000), situated cognition defines learning as "participation in communities of practices" (p.153) and claims that learners conceive of knowledge in terms of the practices they engage in.

Because situated cognition shifts the focus of learning to communities of practice, authentic activities, or ordinary practices associated with the culture in which learning is taking place, become necessary to learners' abilities to transfer knowledge to practices outside of a school setting. The authentic practices do not necessarily need to be real-world applications, but they do require elements supporting active participation in the norms of the discipline. In contrast to the constructivist and information processing views that learning takes place within the learner and knowledge resides in the mind of the learner, the situated cognitive view of learning holds that knowledge is distributed across individuals, artifacts, and tools within a culture. In this perspective, learning includes active thinking and use of available knowledge to do meaningful things in a community, not just integrating new phenomena into existing frameworks within the mind.

## Limitations of a Study as a Research Consideration

Limitations, or potential weaknesses of a study (Creswell, 2003), are inherent in all mathematics education research. Weaknesses can arise from both the design and the implementation of a study, especially in the data collection, analysis, and findings sections of the research. It is important, however, to differentiate between the weaknesses of a study (limitations) and the choices a researcher makes to narrow the scope of a study (delimitations). Nearly every choice made in the course of study can potentially produce limitations (and delimitations too for that matter), so planning for and reporting weaknesses of a study are essential features of quality research. For example, when non-random sampling procedures are required in quantitative research, statistical assumptions of analysis techniques may be violated, which can pose threats to conclusion validity and ultimately limit the inferences that a researcher can make. Sampling procedures can also limit the generalizability of a study's findings; while purposive sampling procedures can provide rich detail about the experiences of a small sample of individuals in qualitative research, they also make generalizing to larger populations tenuous. Bias associated with the researcher, the participants, the theoretical lens of the study, or the collection instruments are all possible limitations of a study and also need to be reported.

Limitations are often closely associated with threats to validity in research. If a qualitative researcher does not triangulate their findings through multiple data collection strategies, for example, their interpretations of the data will be limited by the single-dimensional view provided by their data. In a quantitative study, weaknesses might be attributable to internal validity threats. For instance, in a true experimental design there is a control group and an experimental treatment group. One threat to internal validity can arise if the control group learns of the treatment program for the experimental group (Colosi, 1997). If this happens, the control group may seek to imitate the treatment group (which might lead to a diffusion of the treatment effect), or the control group may even become demoralized from not receiving the treatment (which might lead to decreased motivation to achieve). These kinds of internal validity threats would plainly represent limitations of such a study.

## Validity in Quantitative Research

Validity in quantitative research refers to the accuracy and completeness of an experimenter's findings. One way to look at validity is in how it contrasts with reliability in an experiment. Suppose I conduct an experiment by administering some treatment to a randomly selected sample of participants. I would then collect quantitative observations of the participants using an instrument in hopes of identifying any outcomes attributable to the treatment. Reliability refers to the consistency of my instrument in making the observations. After the observations made by the instrument are collected and analyzed, I could then make any number of inferences and conclusions from the data. Some of these findings could be well supported, but others would not be supported. In this context, Colosi (1997) defines validity as the "strength of our conclusions, inferences, or propositions." Strong experimental validity supports a researcher's ability to conclude that a treatment or intervention affects an outcome (Creswell, 2000).

Four basic types of validity are commonly considered in quantitative social science research: conclusion validity, internal validity, construct validity, and external validity (Creswell, 2000; Colosi, 1997). Conclusion validity is established when a researcher's findings tightly coincides with the statistical assumptions and power found in the design. For example, a threat to conclusion validity would be present if a researcher attempts to fit a simple linear regression model to data that is obviously not normally distributed. If a relationship between a treatment and an outcome is found in a quantitative study, internal validity refers to the amount of evidence supporting a causal relationship (Colosi, 1997). Some threats to the internal validity of a study include maturation of participants during the study, effects of an observation instrument on dependent variables (e.g. taking a pretest might actually improve participants' understanding of number facts), and subjects dropping out of a study. Construct validity is established by designing a study so that the treatments and outcomes are applied and measured according to goals of the research. For example, if one attempts to measure the impact of eating peanut butter sandwiches on student performance in mathematics, but instead gives students peanut butter and jelly sandwiches, then there is a threat to construct validity because the given treatment did not align with the constructs of the experiment. Finally, the ability of an experimenter to generalize their findings to other settings and populations is called external validity. If a researcher attempts to make inferences from collected data to populations not represented in the study, for example, the researcher will encounter threats to external validity.

## Triangulation in Qualitative Research

Triangulation is a term used for efforts made in qualitative research to lend credibility to findings and establish the accuracy of interpretations reported by the author(s). Creswell (2003) reports that triangulation is the easiest and most frequently implemented method of establishing validity in qualitative research. While other methods such as member-checking, rich description, clarification of biases, and reporting of discrepant information are also used, triangulation can infuse confidence into a researcher's perceptions and establish credibility in the minds of the reader's of a study. There are five ways in which one can support their conclusions by examining multiple views of an event: data triangulation, investigator triangulation, theory triangulation, methodological triangulation, and environmental triangulation (Guion, 2002).

Data triangulation is probably the most common way of supporting credibility in qualitative research (Guion, 2002). By collecting multiple forms of data surrounding a phenomenon such as interview transcripts, field notes, and artifacts, an investigator can provide multiple representations of an event. If varied representations point to consistent conclusions, the multiple forms of data improve the credibility/validity of the interpretations. In the case when qualitative observational data is collected in a study, investigator triangulation can be used to provide more than one interpretation of an environment or phenomenon. If two or more researchers independently report very similar interpretations of an event, for instance, the results of the study are supported by investigator validity. Theory triangulation occurs when data is analyzed from more than one theoretical perspective. For example, a researcher might conduct an initial analysis of cognitive data using information processing principles, and then conduct a second analysis using the theory of social cognition. If the separate analyses point to similar conclusions, the validity of the interpretations is supported. Methodological triangulation refers to using mixed quantitative and qualitative methods in a study. This approach supports validity of findings, but also often requires considerably more time and resources to conduct data collection and analysis. Finally, environmental triangulation can be used to support credibility of findings through the altering of environmental conditions that may affect the outcome of a study. For example, suppose one attempts to evaluate a new mathematics curriculum in a school by observing an 8 am class. The researcher might find that students rarely interacted with one another during in-class activities, but the validity of the results would be supported if similar observations were documented for classes at 10 am and 2:00 pm as well.

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